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A MODEL OF OPTIMUM WATER ALLOCATION UNDER IOWA'S PERMIT SYSTEM

by

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CHAPTER ONE: INTRODUCTION

Statement of the Problem

Many of man's activities depend upon the availability of water as one of the earth's resources. In Iowa, virtually every sector of the economy uses water (4, Table 2-A). In the future, industrial, agricultural, and population requirements for water in Iowa are expected to increase (4, Table 4-A) as population and output increase (57, 69). At some point in time, because of the uneven distribution of water supply both seasonally and geographically, requirements for water in Iowa may reach such a level that water becomes a constraining resource, one whose scarcity causes potential production to be foregone. This constraining influence on production could be felt either locally or generally, by individual water users or by groups of users. In such a situation, the manner in which water rights are allocated could have a direct effect on state and local economic fortunes.

In the United States, water has traditionally been allocated by non-market mechanisms. These systems have developed primarily in the customs, legislation, and court decisions of each state. In the State of Iowa, water resources are allocated by a system of water use permits (48). Since the inception of the water permit system in 1957, after a decade of below average rainfall (104), water supplies in Iowa have been relatively abundant.

During this period, all except two permit applications have been granted, and the two which were denied each requested a permit for the drainage of excess surface water.[1] It is therefore impossible to determine, on the basis of its historical performance, how the water permit system would allocate Iowa's water resources if scarcity of those resources began to impose a constraint on the state's economic

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[1] Louis R. Gieseke, Assistant Water Commissioner, Des Moines, Iowa. Data on the history of Iowas water permit system. Private communication. February 9, 1969.

activity. This study addresses itself to the task of predicting, on some basis other than historical performance, the permit system's reaction to a water scarcity and also to the task of developing a method for economic evaluation of this reaction.

The three objectives of this study are as follows: 1) to analyze Iowa's water permit system, constructing an estimate of the system's allocation in times of water scarcity; 2) to construct a model which will yield in specific situations both an optimum water use pattern and values for water in its various uses; 3) to apply the model developed in objective 2 above and the estimate constructed in objective 1 to a specific situation.

Methods and procedures

This study is divided into three procedural phases. The first phase, contained in the first sections of Chapter II, is both descriptive and theoretic in nature. First, water is examined both as a physical and as an economic entity, in an attempt to link the relevant concepts of hydrology and geology to the theoretic framework of economics. Following this in Chapter II, theoretic necessary and sufficient conditions for optimum resource allocation are derived.

The second phase of the analysis, contained in Chapters III through V, examines the origins and general characteristics of the allocative mechanisms currently in use in the United States. To each system are applied the necessary and sufficient conditions developed in Chapter II in order to evaluate each system's recognition of these conditions of optimality. Emphasis is placed on Iowa's permit system in this discussion. As a result of this analysis of the permit system, a specific hypothesis is developed in Chapter IV and a general model is developed in Chapter V under the guidance of this hypothesis. The general model utilizes linear programming to describe the interaction between hydrologic and economic systems, generating approximate values for water optimally allocated among various uses. The results of the model's application can be used in testing hypotheses concerning water allocation under Iowa's permit system.

The third phase of the study is empirical, and consists of application of the linear programming model to two water use situations, one real, the other hypothetical. Chapter VI contains a description of the results of these applications. In Chapter VII, the study and its results are summarized, and conclusions drawn with respect to the applications of the model developed in Chapter V. Recommendations for further research are also suggested in Chapter VII.

CHAPTER TWO: FRAMEWORK FOR ANALYSIS

Iowa's pattern of water use is made up of three dimensions. The first dimension comprises the requirements[1] for water in all its uses, whether as an input to a production process or as a commodity for direct use. The second dimension is Iowa's supply of water, existing both on the surface and underground. The third dimension is Iowa's water permit system, under which rights of use are allocated to particular water users. At any point in time, the pattern of water use in the state, or any local area of the state, is the result of the permit allocation mechanism's interaction with water requirements and water supplies. Conceptual examination of each dimension is a useful prerequisite to discussion of any particular pattern of water use which would result from a scarce water supply.

Water as an Economic Entity

Occurrence

The earth's water supply is circulated by means of the hydrologic cycle, in which water is transferred from land to the sea and back to land (56, p. 8). Precipitation of evaporated seawater in this process accounts for almost the entire supply of fresh water, which occurs either as surface runoff collected in streams and rivers or as underground water collected in aquifers.[2] Units of the quantity of water in the hydrologic cycle are not homogeneous, but are differentiated by the time and location of their occurrence, and by their individual quality characteristics (40, p. 16; 67, p. 1259; 88, p. 7). The physical

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[1] The term "requirements" is used instead of "demands". By definition, demand for a resource is a function of resource price. Under Iowa's water permit system, water has no market price; use of the concept of demand would be imprecise. The distinction between requirements and demands is discussed in a later section of this chapter.

[2] Aquifers are quantities of water occurring in porous strata of rock and soil beneath the earth's surface (56, p. 8).

processes of the hydrologic cycle store, transport, and change the quality of the earth's water, creating and maintaining specific supplies of water throughout the earth.

Supply

The earth's physical supply of water is all water contained in the hydrologic cycle, whether in seas, lakes, or rivers; in the atmosphere or underground. However, portions of the entire physical supply of water are not available for use. At any point in time, use of some portion of the physical supply of water may be prohibited due to restrictions imposed by such social institutions as a legal system (2, p. 18). One such institutional restriction of water use in Iowa is that which prohibits withdrawals when streamflow reaches a certain legally protected minimum (48, sec.455A.1). The amount of water available up to this type of limit is known as the institutional supply (2, p. 18). Further, at any point in time technological limits may make some quantities of water unavailable. The impossibility of reclaiming predictable amounts of atmospheric water when and where they are required (13, pp. 4-7) is an example of a limit placed on water supply by present technology.

Some authors make use of the concept of economic supply (2, p. 18; 16, p. 198; 52, p. 1112; 87, p. 1245). Economic supply is that amount of water which is economically feasible to bring into production. Economic feasibility is determined by the relationship between the cost of acquisition of an additional unit of water and the returns which that unit yields to its users; if returns are greater than costs, use of the additional unit is feasible. The unit cost of acquisition of additional water is influenced by technology, so that if changes in technology decrease the cost, use of previously untapped water may become feasible. Assuming acquisition costs to be constant, an increase in returns to the use of an additional unit of water could increase economic supply. Returns increase if demand for water increases, raising the price which users are willing to pay for an additional unit. These effects of technology and economic conditons on economic supply mean that water supply is not only a function of man's knowledge, but also of man's economic fortunes. Defining economic supply in terms of economic feasibility neatly illustrates the point that technological and economic change may make vast unused water supplies eligible for consideration in meeting existing and potential needs. The extent of these potential water supplies would be dependent upon the existence of any institutional or technological limit on physical supply.

Water supplies may be characterized as either stock or flow supplies. Kelso (52) defines stock supplies as those whose physical quantity does not increase appreciably over time; therefore, each rate of use of a stock resource diminishes some future rate of use. In defining a flow supply, Kelso points out that different units of the supply become available at different times, and that present flow does not diminish future flow. Therefore, it would be possible to maintain use of a flow resource indefinitely if flow continues. The hydrologic cycle, precipitation, surface runoff, and streamflow are examples of flow supplies of water, while an aquifer which recharges at a very slow rate could be considered a stock supply, fixed in magnitude.

Water use classification

The uses to which water resources can be put are myriad, perhaps as numerous as man's activities. A number of different schemes exist whereby these uses can be classified. One such device classifies water use by the final product, process, or activity of which water is a part, under the two general headings of production and consumption uses. Water uses in industry, mining, and agriculture are production uses (87, p. 1245), while such uses as human consumption and recreation are consumption uses (87, p. 1245; 16, p. 198). This method of classification is useful for the economist, for the same categories can be applied to water as an input or commodity in constructing demand relationships.

Some water uses, both production and consumption, may be designated as consumptive. In the traditional riparian definition,[3] a water use is consumptive if the quantity of water in the watercourse is diminished by such use (1, p. 104; 43, p. 7; 90, p. 272). However, defining consumption in terms of quantity alone ignores other important ways in which a use may be consumptive. As an example, consider an industrial water user who returns to the watercourse all the water he withdraws, but returns it laden with the by-products of his production process. If a downstream user must treat intake water to remove these industrial pollutants, the second user is restoring quality utility which the upstream user consumed. It is therefore important to consider depletions in the utility which units of water in a watercourse possess, as well as depletions in quantity in the watercourse, when considering consumptive use.

Source depletion is also an important characteristic of those water uses which withdraw water from a stock supply. Since present rates of use of stock water supplies directly affect future rates of use, allocation decisions must be made inter-temporally, as well as among uses and users.

Finally, economic theory provides one further classification of water use by enabling the relationships between water uses to be characterized as complementary, competitive, or neutral (87, p. 1246; 76, p. 162). According to Timmons(87), water uses are complementary if allocation to one use increases net benefits accruing to water in another use, while a competitive relationship exists if one water use restricts net benefits available from another. If net benefits available from different uses are not affected by allocation to one use or another, the relationship is one of neutrality. Any consumptive use of water is competitively related to most other uses of that water, since

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[3] The riparian doctrine is that legal system under which water rights are allocated in most of the thirty-one eastern states. Further discussion of the riparian doctrine is contained in a later section of this chapter.

allocation to a consumptive use generally does not permit further use of the water without at least restoring the utility which was consumed. A use which is consumptive with respect to quality impairment may not interfere with another use which requires low water quality, but this relation is at best neutral. Water used in hydroelectric power generation complements the use of that water for recreation, since power generation ideally requires a constant head of water, which would provide a constant reservoir depth for swimming, boating, or fishing. Use for power generation would be neutral with respect to downstream uses, since only the energy head of the elevated water is used in generating power, not necessarily impairing quality or quantity. Complementary uses are not the major concern in allocation, nor are neutral uses, for a unit of water allocated to one use does not decrease or preclude the benefits available from that unit of water in another use if the two uses are neutral or complementary with respect to each other. However, problems arise when allocation decisions must be made among competing uses for a water supply, since only one of the competing uses can realize the benefits accruing from use of the water.

In summary, the methods of classifying water use described above characterize each water use according to the product or activity of which it is part, and designate each use as consumptive, nonconsumptive, or source-depleting. Further, sets of uses are characterized as complementary, neutral, or competitive. Such a scheme coincides with concepts advanced by Snyder(82), who holds that uses should be categorized based on the concept of utility addition. Each use would be considered a conversion of water in an economic process such that not only quantity, but also time, place, and quality utility may be modified. Uses would then be identified according to their effect on Pareto[4] optimality, which embodies the idea of interaction among water users.

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[4] For a comprehensive discussion of the conditions of Pareto optimality, see (78, pp. 148-188).

Demands and Requirements

In discussions of non-market allocation of water rights, it is important to distinguish between demands and requirements for water. The distinction between these two terms as used in this study can be illustrated by a discussion of the concept of water resource demand.[5] Demand for a resource is of two types, direct and indirect. For uses in which water is a factor input to be transformed into some product, demand is indirect. For such uses as drinking or bathing, water, or the utility which it possesses, is directly consumed; for these uses, demand is direct. Demand of both types is expressed for units of water of a particular quality at a particular time and location.

Direct and indirect demand are both based upon physical relationships. Direct demand by a water user is based upon the relationship between that individual's consumption of alternative amounts of water and the utility which he derives in consumption. This relationship between utility and consumption is expressed by the concept of the utility function. Indirect demand by a firm for water as a factor input is based upon the firm's production function, a technological relationship describing the transformation of a set of factor inputs into some product.

Demand for water, however, is dependent upon more than the physical relationships described above. The price of the water resource is an important component of demand, for in their purchases of water, direct demanders are constrained by a finite income, while indirect demanders are constrained by a finite revenue from the sale of the product of which water is a part. Thus, if water has a market price, the amount users are able to buy depends upon the level of market price and the amount of money available for the purchase of water.

Another important component of demand is the set of prices of other goods, particularly those which are either complements or substitutes for water as a commodity or factor. The amount of water

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[5] General discussion of the concepts of demand can be found in (32, pp. 26-42).

which a user is willing to buy varies directly with the prices of substitute goods and inversely with the prices of complementary goods (32, pp. 54-60).

In summary, demand for water is initially derived from physical relationships. Direct demand is derived from the consumer's utility function, indirect demand from the firm's production function. In addition, demand for water is dependent upon the price of water, the prices of other goods which may complement or be substituted for water, and consumer's income or firm's revenue.

In this study, the term "requirements" is used to refer to demand in situations where there is no market price for water. The term is used in this way for two reasons. First, where water has no price, if income or revenue and the prices of other goods are held constant the requirement for water derives solely from the consumer's utility function or the firm's production function. This relationship is physical, not economic; to label it demand would be imprecise and misleading. Second, in situations where water supply is insufficient to satisfy all requirements and where water has no market price, the requirements of alternative uses do not reflect the opportunity cost of water. Water has an opportunity cost if allocation of water to one use requires that production be foregone in other uses (32, p. 164), as is true in situations of insufficient supply.

In situations where market allocation of water rights is prohibited or restricted by a legal system, the concept of demand is of limited usefulness for the reasons cited in the above discussion. In addition, the applicability of a microeconomic industry or market analysis to water allocation is limited by at least the following factors:

a) many decentralized users, such as farms, industry, and non-profit water organizations, are self-supplied (16, p. 200; 18, p. 3). In 1950, 99 per cent of agricultural irrigation and 97 per cent of industrial use were self-supplied (16, p. 200). Allocation decisions in these cases are internal, and not expressed in the market place.

b) As a commodity or factor, water is not homogeneous; for each demand, differentiated by quality, time, and location requirements, a "market" could exist. Thus, there would be no reason to expect a single price for water.

c) Forms of ownership of rights to use of water are diverse.[6] Among the various legal and administrative systems of water allocation in the United States, and within each system, a variety of restrictions have been placed on the free use and transfer of water. Without freedom to transfer commensurable rights to use a product, traditional market analysis is crippled.

d) Forms of payment are also diverse, and possibly are not based on a concept of market price (10, p. 37; 16, p. 198; 65, p. 2); *ad valorem* taxes have been the most popular mode of payment (10, p. 38).

For these reasons, it appears that the problems which need to be considered are more those of organization and management of self-supplying firms than of a traditionally defined industry (16, p. 201). However, even in their limited capacity, market concepts will prove useful in analysis of water allocation problems, since competition for water could develop among self-supplying firms.

Theoretic Conditions for Optimum Resource Use

It is not unreasonable to assume that the appropriative doctrine, the riparian doctrine, and Iowa's permit system (all to be discussed in the next chapter) were designed to be optimizing institutional mechanisms (94, p. 6). In this section, theoretic necessary conditions for optimum water resource use are derived and, under the assumption above, are applied to each of the three allocation mechanisms to show how allocation under each system can differ from the optimum. Necessary conditions for optimum resource use can easily be shown using a

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[6] A summary description of these forms of ownership is contained in a later section discussing water law.

classical optimization method, the technique of Lagrange multipliers. The general maximization case of this technique, utilizing inequality constraints, treats problems of the form[7]

$$\begin{aligned}
&1) \quad \max Z = f\ (x), \text{ satisfying} \\
&\qquad g_i\ (x) \leqslant b_i \qquad i = 1, \ldots, u, \\
&2) \quad g_i\ (x) \geqslant b_i \qquad i = u + 1, \ldots, v, \\
&\qquad g_i\ (x) = b_i \qquad i = v + 1, \ldots, m,
\end{aligned}$$

where X is an n-component vector. Adding slack and surplus variables, the original constraints are equivalent to

$$\begin{aligned}
&\qquad g_i\ (x) + x_{si} = b_i \qquad i = 1, \ldots, u, \\
&3) \quad g_i\ (x) - x_{si} = b_i \qquad i = u + 1, \ldots, v, \\
&\qquad g_i\ (x) \qquad\quad = b_i \qquad i = v + 1, \ldots, m.
\end{aligned}$$

The corresponding Lagrangian function is

$$4) \quad F\ (x, x_s, \lambda\ ) = f\ (x) + \sum_{i=1}^{u} \lambda_i \left[b_i - x_{si} - g_i\ (x)\right] + \sum_{i=u+1}^{v} \lambda_i \left[b_i + x_{si} - g_i\ (x)\right] + \sum_{i=v+1}^{m} \lambda_i \left[b_i - g_i\ (x)\right] .$$

In order for f (x) to take on a maximum at x_o, the following necessary conditions must hold:

$$\begin{aligned}
&5) \quad \frac{\partial F}{\partial x_j} = \frac{\partial f(x_o)}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i(x_o)}{\partial x_j} = 0 && j = 1, \ldots, n; \\
&\qquad \frac{\partial F}{\partial x_{si}} = -\lambda_i = 0 && i = 1, \ldots, u; \\
&\qquad \frac{\partial F}{\partial x_{si}} = \lambda_i = 0 && i = u+1, \ldots, v; \\
&\qquad \frac{\partial F}{\partial \lambda_i} = b_i - x_{si} - g_i\ (x) = 0 && i = 1, \ldots, u; \\
&\qquad \frac{\partial F}{\partial \lambda_i} = b_i + x_{si} - g_i\ (x) = 0 && i = u+1, \ldots, v; \\
&\qquad \frac{\partial F}{\partial \lambda_i} = b_i - g_i\ (x) = 0 && i = v+1, \ldots, m.
\end{aligned}$$

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[7] Equations (1) through (5) are taken from the excellent discussion of constrained optimization in (37, pp. 69-71).

Sufficient conditions for f (x) to be a maximum at x_o are satisfied if the second total differential of f (x_o) is negative (41, p. 272, note 1), a condition which is fulfilled if f (x) is concave. For this analysis, it will be assumed either that second-order conditions are fulfilled, or that f (x) is concave, at least in the range relevant to analysis.

Having derived the desired necessary conditions in the general case alone, an objective function and constraint equations can be specified, relevant to water resource use, and particular necessary conditions derived for optimum resource use. The following assumptions are made in order to simplify and restrict the analysis to the considerations of this study:

a) there are n perfectly competitive firms using water in amounts x_j, $j = 1, \ldots, n$, from a homogeneous supply fixed at $\bar{x}$;[8]

b) each firm's production function, in truncated form,[9] can be written as $Q_j = f_j(x_j)$, where Q_j is the output of the jth firm's product;

c) resource use decisions are made under an aggregate objective function, expressed in terms of total output of the n firms using water.

If the objective is to maximize total value of production, expressed as

$$6) \quad \max Z = \sum_{j=1}^{n} P_j Q_j = \sum_{j=1}^{n} P_j f_j (x_j),^{10}$$

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[8] Such a group of firms corresponds to a "watershed firm," a concept elaborated and utilized by Timmons. See (85).

[9] In the truncated form of the production function, all other inputs are assumed to be held constant. The necessary conditions for optimum resource use with respect to any single input are identical whether other inputs are constant or variable.

[10] The objective function can take this form only if output price is constant regardless of the level of output. This condition is fulfilled in the assumption of perfect competition.

where P_j is the price of the jth product, subject to the constraint

7) $\sum_{j=1}^{n} x_j \leqslant \bar{x}$,

the necessary conditions for maximum Z at x_j^*, ($j = 1, \ldots, n$), after adding a slack variable to the constraint, are

8) $\frac{\partial F\ (x^*,\ x_s^*, \lambda^*)}{\partial x_j} = P_j \frac{\partial f_j\ (x^*j)}{\partial x_j} - \lambda^* = 0 \qquad j = 1, \ldots, n;$

9) $\frac{\partial F\ (x^*,\ x^*s, \lambda^*)}{\partial x_s} = - \lambda^* = 0;$

10) $\frac{\partial F\ (x^*,\ x^*s, \lambda^*)}{\partial \lambda} = \sum_{j=1}^{n} x_j + x_s - \bar{x} = 0.$

Three important relationships are contained in these necessary conditions. First, from equation 8, it can be seen that

11) $P_j \frac{\partial f_j\ (x^*j)}{\partial x_j} = P_i \frac{\partial f_i\ (x^*i)}{\partial x_i} \qquad i, j = 1, \ldots, n.$

$\frac{\partial f_j\ (x^*j)}{\partial x_j}$ is the marginal physical product of x in the production of Q_j, and $P_j \frac{\partial f_j\ (x^*j)}{\partial x_j}$ represents the value of marginal product (vmp) of x in the production of Q_j. Equation 11 defines the critical condition that, for optimum resource use, the vmp of the resource must be equal in all its uses.

Second, it can be shown that the following relationship holds:

12) $\lambda^* = \frac{\partial Z^*}{\partial \bar{x}}$ (37, p. 73).

From this relationship, λ^* can be defined as the shadow price of water and is equal to the value of an additional unit of water. It is apparent from equation 8 that the unit value of water must be equal in all its uses. The third relationship follows from equation 9, and is

13) $x^*_s \lambda^* = 0$ (37, p. 72),

which means simply that if $x^*_s > 0$, $\lambda^* = 0$; if $x^*_s = 0$, $\lambda^* \neq 0$. x_s is a slack variable, and is positive only if the supply of water, $\bar{x}$, is not fully utilized. Therefore, from equation 13, if water is abundant and some of the supply remains unused, then the shadow price, or unit value, of water is zero.[11]

Possible Divergences from the Theoretic Optimum

Assuming that the second-order conditions noted above are fulfilled, allocation of water such that vmp is equal in all uses implies that the value of the objective function for the watershed firm is a maximum. Whether this maximum is also optimum with respect to larger planning units, such as the basin, state, or nation, depends on the equality of cost and benefit to the watershed firm (marginal private cost and benefit) with marginal cost and benefit to the larger planning area (marginal social cost and benefit). External economies or diseconomies (5, pp. 368-371; 32, pp. 391-394) may be present which cause marginal private cost and benefit and marginal social cost and benefit to diverge.

External economies and diseconomies are of two types: production and consumption (5, p. 369). Water pollution is a pertinent example of an external production diseconomy, in that pollution of water at one point on a stream incurs cost to any downstream user who must resort to substitute supplies or treat the water prior to his own use. Interaction between the production function of the downstream user and the upstream polluter implies that the downstream user must expend more inputs to produce the same output possible with unpolluted water. In this case, the marginal private cost of the polluting firm is less than its marginal social cost, if the pollution it causes is considered to be a negative portion of its output (78, p. 187).

An external production economy, conversely, occurs when marginal social benefit exceeds marginal private benefit (5, p. 369). This type of externality would occur if an upstream water user applying

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[11] Under the same assumptions employed above, plus the assumption that each water user is a profit maximizer, it can be shown that each producer will employ an input until its vmp is equal to the input price (32, p. 309). Therefore, at the optimum, decentralized resource allocation and allocation under an aggregate objective function are theoretically the same, and λ can be considered the market clearing price.

water to a cooling process discharged heated water into a stream from which a downstream user requiring heated water could withdraw. The increase in temperature from the upstream user's operation allows the downstream firm to expend fewer inputs in producing its output, making marginal social benefit greater than marginal private benefit.

External economies and diseconomies which arise from one individual's consumption are defined in much the same way as production externalities. The major difference is that any divergence between marginal social values and marginal private values arises as the result of consumption rather than production.

An important point with respect to external effects is made by Pigou (74, p. 183). He states that the existence of an externality is contingent not only upon the existence of interdependence between two or more producers, but also upon the lack of compensation for benefits or injuries resulting from this interdependence. This point qualifies the statement that external effects tend to cause misallocation of resources (5, p. 371). However, if costs and benefits can be measured, compensation is a remedy which can be applied to enhance optimum resource allocation.

Having established in this chapter a background of concept and theory, Chapter III will discuss the allocative mechanisms under which water resources are controlled in the United States. Following this discussion, the allocative systems will be examined from the theoretic point of view established in this chapter.

CHAPTER THREE: LEGAL SYSTEMS OF WATER RESOURCE ALLOCATION

In the United States, most of the productive resources and factor inputs of the economy are allocated by market processes. Water, however, is one resource which has traditionally not been distributed by a market mechanism. Instead, a number of complex legal allocation systems have developed in the United States, evolving from customs, legislation, and court cases in each state (46, p. 868).

Possibly the most important contributing factor in the growth of non-market allocation systems is the fact that water is a migratory resource; the flow of water does not respond to the delineation of property boundaries and political units. According to Harl (38, pp. 19-20), property rights in such a fugitive resource are generally less certain and unequivocal than rights in other property. Two factors which create uncertainty in a water right are a) the possibility that the water to which the right pertains will not be available, due to variability in physical supply; and b) the possibility that the water may be consumed by an upstream user. Because of the inherent uncertainty in a water right, the quantity of water which the right holder will have available for use is indeterminate. This quantity could vary from zero to the full amount defined by the right, depending upon hydrologic conditions and the exercise of any prior rights.

In turn, uncertainty of quantity leads to a similar uncertainty about price, since the unit price of a commodity generally varies with the quantity demanded or supplied. Establishment of a market in water rights could be inhibited by the lack of a clear price for water.

An additional obstacle to the establishment of a market for water is found in the fact that the use of a unit of water may cause changes in the hydrologic system where the water was used and in the system from which it was drawn if the two systems are not the same. Examples of such concomitant changes are a change in water quality downstream from the point of use or a change in the conditions in an aquifer due to heavy withdrawal at one point. Such external effects as these may have sutstantial impact on parties who would not ordinarily be represented in any market transaction from which the external effect results. The obvious avenue for redress for such damages resulting from a transfer of water rights would be the courts. In this way, the establishment of legal precedents and principles of water allocation would be expected to accompany competition for water rights, if external effects resulting from transfers of rights are significant.

As a result of the unique character of water, several general legal systems of water allocation have developed in the United States, each adapted to the peculiarities of the region where it is practiced. In general, surface water allocation in the thirty-one eastern states is regulated under the riparian doctrine (94, p. 5), while the seventeen western states have developed a doctrine of appropriative rights (94, p. 5). In several states, administrative allocation systems, such as Iowa's water permit system, have been proposed or enacted (94, p. 5).

In order to provide a framework for evaluating the degree to which the appropriative doctrine, the riparian doctrine, and Iowa's water permit system recognize the necessary conditions for optimum resource use, each legal system will be examined in the role of an optimizing mechanism. Adoption of this point of view provides specific direction to the examination of each legal system, for a viable optimizing mechanism necessarily possesses the following characteristics:

a) a clear, identifiable objective;

b) provision of a mechanism which can measure and compare selected parameters for decision making; and

c) identification of a set of measurable parameters upon which alternative courses of action can be compared.

The following discussion identifies and examines these characteristics in each of the three legal allocation systems listed above.

The Doctrine of Prior Appropriation

The so-called appropriative doctrine is based on Mexican and Spanish rights, developed in Utah by Mormon settlers and in California by miners after the discovery of gold in that state in 1848 (1, p. 104; 46, p. 867). The doctrine is followed chiefly in the seventeen states west of the Missouri River and in Alaska (1, p. 104; 38, p. 24). These states are characterized by broad similarities in their water allocation systems (46, p. 873): the water resources of the state are under public control by statute (94, pp. 19-20), with management placed in the hands of state officials, and statutory or administrative declaration is made concerning waste and beneficial use of water (46, p. 873).

An appropriative right is based on the "law of the first taker" (55, p. 28), the principle which governed mining rights during the pioneer days. Indeed, the first beneficial uses of water noted in this doctrine were in placer mining and gold refining (90, p. 279). The right has also been called "first in time, first in right" (1, p. 104). Whoever first took possession of water and put it to a beneficial use retains the right to use that water. It is upon this claim in history that an appropriative right is based. The right is defined in priority, quantity, period of use, and point of diversion (1, p. 105; 15, p. 256; 38, p. 27; 43, p. 22; 55, p. 28).

There are two elements of an appropriation (1, pp. 104-105). First, there must be an actual diversion of water, with the intent to apply it to some beneficial use. Second, the water must be applied to that use or some other beneficial use. The concept of beneficial use, as expressed in these elements, is central to the appropriative doctrine (90, p. 277), as evidenced by the maxim that "beneficial use is the basis, the measure and the limit of the right to use water...." (90, p. 277).

A few states list specific uses as beneficial, including domestic, municipal, stock watering, irrigation, manufacturing, and mining (1, p. 106; 38, p. 24; 90, p. 227). States have not, however, provided general definitions of beneficial use (1, p. 106; 90, p. 277), and some opinions hold that the question must be decided separately in each case

(1, p. 106; 90, p. 277). In all states, a use must not only be beneficial to the user, but must also be reasonable with respect to other uses and future demands for water (90, p. 284). The reasonable use criterion is apparently intended to insure that a privately beneficial use is not also socially detrimental. Reasonable use is defined in terms of relative economy and waste in intended uses (90, p. 284).

Centralized state control over appropriation has developed in almost all the western states (1, p. 105), and orders of preference among uses have evolved (39, p. 26; 90, p. 285). There is little general agreement among states on order of preference, except that man's survival needs, including water for drinking, bathing, and sanitation, come first, and navigation and water-based transportation are last (90, p. 286). Other uses, such as irrigation, mining, and manufacturing, vie for the middle ground of priority (90, p. 286). Some states require state officials to grant priorities among appropriations according to statutory preference ranks, while other states allow state officials to exercise discretion in granting priorities (90, p. 285). In both cases, preferences are based on relative benefit (90, p. 285). Under these preference rankings, water may be reallocated in one direction only along the preference scale, from less preferred uses to more preferred uses (90, p. 285). Rights may also be lost by abandonment, forfeiture, or action against an adverse use (1, p. 108).

In general, apart from a transfer of ownership of the land on which the right is based, transfer of an appropriative water right to another type of use or point of diversion is difficult. In some states,[1] a water right may not be transferred from either the original use or the original point of use (94, p. 69). In other states, the party desiring the transfer must prove that no damage will occur to other users of the water supply affected by the transfer (40, p. 22). To prevent loss of return

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[1] Notable examples are Arizona and Wyoming (40, p.22).

flow by transfer or rights, the general rule has been established that only the amount of consumptive use may be transferred (40, p. 22).

The Riparian Doctrine

In the thirty-one states east of the Missouri, and to a degree in some western states, a system of riparian water rights has developed, from roots in English common law (1, pp. 99-100; 46, p. 867). Central to the riparian doctrine is the concept of riparian land as that land which borders the course of a stream or underground watercourse (1, p. 100; 38, p. 23; 43, p. 6; 55, p. 26). The right of a riparian owner, which exists as a consequence of ownership of riparian land, gives him the use of water flowing in a watercourse which abuts his land, providing the water is returned, unimpaired in quantity and quality, except for impairment inseparable from reasonable use (1, p. 100; 38, pp. 22-23; 43, p. 6). The right is a modification of two legal concepts (38, p. 21; 90, p. 273). The first, the natural flow theory, grants a riparian owner the right to a "...natural condition of flow." (38, p. 21). The second concept, that of reasonable use, was imposed upon the earlier theory in order to allow uses which are consumptive (38, p. 21).

A riparian right is based on the nature of the source and the nature of the use (43, p. 4). Sources are defined as 1) diffused surface water, 2) surface watercourses, 3) underground watercourses, and 4) underground percolating water (43, p.4). Riparian owners may use these types of waters, except as limited by the rights of other riparians and restrictions based on certain categories of use (43, p. 4).

Uses are divided into two major categories, natural and artificial (90, pp. 273-274). Upstream riparian users may, if necessary, consume all the water in a surface watercourse for natural uses (38, p. 22; 43, p. 4; 90, p. 274), which include domestic use and watering an ordinary number of livestock (38, p. 22; 43, p. 4; 90, p. 274). Artificial uses, such as irrigation, industrial use, and municipal water systems (43, p. 8; 90, p. 274), are subordinate to natural uses (43, p. 8). Rights of all riparian owners with respect to actificial uses are coequal (1, p. 101; 17,

p. 877; 43, p. 7; 90, p. 274), and allocation decisions are based on relative reasonableness (43, p. 7). Determinations must be made in each case of how reasonable an intended artificial use will be (1, p. 101; 38, p. 22; 43, p 9; 90, p. 283). No rules of reasonable use have been laid down by courts because what is reasonable in light of the equal rights of other riparians changes as physical, demographic and economic conditions change (1, p. 101; 90, p. 283).

In general, riparian rights are restricted to lands contiguous to the watercourse (1, p. 101; 38, p. 23; 43, p. 9) and contained in the watershed (1, p. 101; 43, p. 9). There are, however, exceptions to both these principles. In some cases, rights to use water have been transferred from riparian to non-riparian lands. In a number of these cases, the riparian land and the non-riparian land were held by different owners. The remainder of the transfers were from riparian land to non-riparian land held by the same owner (59, pp. 55-56; 94, p. 65). In Ohio, a city which is riparian is entitled to take water for use by its residents, even though they may be located outside the watershed (94, p. 16).

As a general rule, riparian rights are not lost by nonuse (1, p. 103), since these rights are not based upon use, but upon ownership of a particular type of land. Only adverse use or eminent domain proceedings can destroy a riparian right (1, p. 103).

Doctrines Governing Underground Water Supplies

The two underground water sources differentiated in law are underground watercourses and percolating groundwater (43, p. 4; 44, pp. 232-233; 47, p. 293). This distinction has been criticized by hydrologists as inapplicable, but continues to be observed in law (44, p. 233; 47, p. 294). Underground watercourses are governed by the legal system operating for surface watercourses in the area (44, p. 233; 47, p. 244). Percolating groundwater, which is water underground and not moving in a reasonably defined course (43, p. 9; 44, p. 233; 47, p. 274), is controlled by one of the following three doctrines. One, the

English rule, or the common-law doctrine, grants absolute ownership of the underground water to the overlying landowner (44, p. 233; 47, p. 294). The freedom of use associated with this doctrine led some jurisdictions to apply another doctrine, the American rule of reasonable use (43, p. 9; 47, p. 294), which recognizes the right of the overlying landowner but restricts his use of percolating groundwater with respect to waste or transportation to a distant use (43, p. 9; 44, p. 234; 47, p. 295). The third doctrine controlling percolating groundwater is that of "correlative rights" (44, p. 234; 47, p. 295), found chiefly in California. Under this doctrine, the rights of overlying owners are coequal for reasonable use; any surplus beyond reasonable use by these landowners may be appropriated for use on non-overlying lands, and in shortage situations the available supply is apportioned among overlying landowners in proportion to their reasonable needs (44, p. 234; 47, p. 295).

It appears that the riparian and prior appropriation doctrines, although legally dissimilar, have similar objectives. Each system seeks to provide a mechanism for orderly allocation of water rights, according to the parameters of reasonable and beneficial use. The decision-making mechanism in both doctrines is one of adjudication guided by legal principles and precedents. However, in both the appropriative and riparian doctrines, these principles may act to restrain transfer of water to more beneficial uses. The economic significance of these restraints will be examined in a later section of this chapter.

Administrative Allocation: Iowa's Water Permit System

Seven of the states under the riparian doctrine[2] have proposed or enacted programs which modify the riparian doctrine (15, p. 252; 29, p. 237). In some instances, as riparian concepts are modified they are replaced with appropriative concepts (29, p. 252). In other instances, the trend has been toward grants or permits, administered by a central

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[2] The seven states are Florida, Iowa, Maryland, Minnesota, Mississippi, North Carolina, and Wisconsin (94, p. 5).

state authority (29, p. 252; 38, p. 27). Of these modifications, the one most important to this study is that which has been made in Iowa. The Iowa water permit system is similar to earlier proposals in other states (43, pp. 24-25), notably Wisconsin, Minnesota, and North Carolina. This study focuses on the Iowa system. Where significant differences exist between the Iowa system and proposals in other states, these differences are noted.

Iowa's permit system, enacted in 1957, is defined by statute (48). Administrative decisions have been made in implementing the permit system which have become, operationally, a part of the mechanism,[3] but the statute which created the permit system nonetheless constitutes its basic framework. For this reason, the analysis in this section will be based mainly on an examination of the provisions in the statute.

Objective of the water permit system

It is difficult to specify an objective for the permit system as an allocative mechanism. The following appears in the statute which creates the Iowa water permit system:

> "It is hereby declared that the general welfare of the people of the state of Iowa requires that the water resources of the state be put to beneficial use to the fullest extent of which they are capable, and that the waste or unreasonable use, or unreasonable methods of use, of water be prevented..." (48, sect. 455A.2)

The statement specifies that each use be reasonable, beneficial, and not wasteful, but what constitutes the optimum degree of each is open to some difference of interpretation. The requirement that Iowa's water resources "... be put to beneficial use to the fullest extent of which they are capable..." (48, sect. 455A.2) could be interpreted in at least two distinct ways. First, the statement could mean that a maximum amount of water should be allocated to those uses which can be classified as beneficial. Alternatively, the statement could mean that the state's

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[3] An excellent review of permit system operations between 1957 and 1967 can be found in Hines (43).

water resources should be allocated among all uses such that total benefit is a maximum. The two interpretations imply different conditions from the point of view of economic theory. Two similar interpretations could follow from the statement that declares control of the state's water resources to be in the state, in order "...to effectuate full utilization..." (48, sect. 455A.2), which could imply either use of a maximum amount of water, or allocation of the state's water resources such that maximum benefit per unit is achieved. Furthermore, there is no indication in the objective statement of whether the general welfare of the people of Iowa is to be maintained, increased, or maximized with respect to water use.

A set of definitions is contained in the statute (48, sect. 455A.1). Most of the terms with which the statute is constructed are defined, with the immediate exception of the terms "general welfare" and "reasonable use." Reference to the following two definitions assists in making the policy statement more specific:

> "'Beneficial use' means the application of water to a useful purpose that inures to the benefit of the water user and subject to his dominion and control but does not include the waste or pollution of water;" (48, sect. 455A.1)
>
> "'Waste' means (a) permitting ground water or surface water to flow, taking it or using it in any manner so that it is not put to its full beneficial use, (b) transporting ground water from its place of use in such a manner that there is an excessive loss in transit, (c) permitting or causing the pollution of a water bearing strata through any act which will cause salt water, highly mineralized water, or otherwise contaminated water to enter it;" (48, sect. 455A.1)

Imposing these definitions on the statute's stated goal (48, sect. 455A.2) facilitates a slightly more precise paraphrase of the statute's objective: the general welfare of the people of Iowa requires that the state's water resources be put to fully beneficial uses to the fullest extent of which they are capable; these uses should be reasonable and cause no pollution or excessive loss in transit of the state's water resources. This restatement of the statute's objective still does not indicate whether maximization is desirable, or which variable or combination of the three variables (general welfare, benefit, and quantity allocated) is to be considered the goal of the system. As a method of selecting among alternative water allocations, Iowa's water permit system has no adequately specific objective statement.

The water permit system's administrative mechanism

To implement its stated policy, the statute creates and vests authority in the Iowa Natural Resources Council (48, sect. 455A.2-.3). Composed of nine members, the Council is charged to "...establish a ...comprehensive state-wide program for the conservation, development and use of the water resources of the state" (48, sect. 455A.17). The statute declares the water occurring naturally within the state to be public wealth of the people of Iowa (48, sect. 455A.2), and gives the Iowa Natural Resources Council jurisdiction over public and private waters in the state. The Council is directed to study and survey the state's water resources and their relation to problems in agriculture, industry, conservation, health, and stream pollution. Recommendations are to be made for further development, utilization, protection, and preservation of these water resources.

The statute provides for the selection of a water commissioner and one or more deputy commissioners, who serve at the Council's pleasure. The commissioner tries fact questions in processing permit applications, and conducts hearings on each application (48, sect. 455A.9).

Although jurisdiction of the Council is broad, not all uses are to be regulated. The following definitions partially limit the scope of regulation:

> "'Depleting use' means the storage, diversion, conveyance, or use of any supply of water which might impair rights of lower or surrounding users, or might impair the natural resources of the state or might injure the public welfare if not controlled;" (48, sect. 455A.1)
>
> "'Regulated use' means any depleting use except a use specifically designated as a nonregulated use;"[4]

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[4] (48, sect. 455A.1). Only irrigation uses are regulated under the permit systems of Wisconsin and North Carolina (29, pp. 239, 244).

> "'Nonregulated use' means the use of water for ordinary household purposes, use of water for poultry, livestock and domestic animals, any beneficial use of surface flow from rivers bordering the state of Iowa, or use of ground water on islands or former islands situated in such rivers, existing beneficial uses of water within the territorial boundaries of municipal corporations on May 16, 1957, except that industrial users of water, having their own water supply, within the territorial boundaries of municipal corporations, shall be regulated when such water use exceeds three per cent more than the highest per day beneficial use prior to May 16, 1957, and any other beneficial use of water by any person of less than five thousand gallons per day;" (48, sect. 455A.1)

A permit is required for all regulated uses as defined above. In addition, diversions of water from the surface to underground which existed prior to May 16, 1957 are exempt if they cause no pollution, but such diversions begun after that date must have a permit.[5]

Thus, a wide range of regulation is established, and the permit instrument is created to control allocation to uses throughout the range. The permit is the council's written authorization for use, limited "...as to quantity, time, place, and rate of diversion, storage or withdrawal..." (48, sect. 455A.1). The procedure for securing a permit is initiated by written application to the Council. The application, accompanied by a fifteen dollar fee, describes the intended beneficial use (48, sect. 455A.19).

Upon receipt of an application, the Council investigates the effect of the intended use upon other interests in the area (48, sect. 455A.18). and the water commissioner sets the date and location of a hearing (48, sect. 455A.19). A notice of the hearing, describing the intended use, must be published in the county of the proposed use prior to the hearing date. Copies of the notice are sent to officials in other interested state agencies, including the Conservation Commission, the Public Health Service, the Iowa Geological Survey, and the Iowa Development Commission (48, sect. 455A.19).

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[5] (48, sect. 455A.25). In Minnesota, no use originating within a municipality requires a permit (29, p. 241).

At the hearing, interested parties may appear and present evidence (48, sect. 455A.19). On the basis of due investigation and testimony, the commissioner determines whether the intended use will be detrimental to either the public interest or the interest of any property owners with prior or superior rights. If not, a permit is granted (48, sect. 455A.20). Aggrieved parties may appeal the commissioner's decision to the Council within thirty days, and be granted a hearing before the director (48, sect. 455A.19).

Definition of a set of decision making parameters

The objective statement discussed earlier indicates that a use should be beneficial, reasonable, and not wasteful (48, sect. 455A.2). A beneficial use could be defined as one in which marginal benefit to the user is positive, but the statute specifies no measurable variable to represent this benefit. Waste is said to occur if any use is less than fully beneficial, if there is excessive loss in transporting groundwater, or if pollution of any groundwater is allowed through the introduction of any contaminated water into the supply (48, sect. 455A.1). Beyond the reference in this definition, what constitutes pollution is not specified, but water quality standards have been formulated under separate authority (50).

It is possible to classify uses as beneficial or not, based on the qualifications of waste and pollution. Deciding among alternative beneficial uses, however, requires that the alternatives be ranked. The statute provides that applications are to receive consideration based on date of application, and that certain uses existing prior to May 16, 1957 will be granted priority according to date of use (48, sect. 455A.21). In addition, the statute states that if no detriment to public or private interest can be found in an intended use, the commissioner "...shall grant a permit..." (48, sect. 455A.20) for that use. If not all uses can be satisfied, these standards and priorities may not aid in achieving the statute's objective, for they do not provide assistance in measuring relative benefit.

The importance of beneficial use is reinforced by statements granting the Council authority to issue permits to these uses (48, sect. 455A.22) and declaring that in the disposition of applications, the standard is to be beneficial use (48, sect. 455A.21). Relative benefit would then seem to be the critical factor in ranking alternative allocations, but since benefit is not defined so as to allow measurement, comparison among uses on this basis is not possible unless administrative judgments are made. (The costs of waste and pollution are measurable in theory, but since any use for which these costs are positive is not permitted, measurement is irrelevant.)

On the basis of the preceding discussion, it is apparent that the statute is unclear or incomplete on two vital points. First, its objective is not stated in unique, measurable terms. Second, the criterion of benefit from use, on which comparison and allocation among competing uses would be made, is not defined in measurable terms. Therefore, comparisons among uses are not possible. In a situation of inadequate supply, the permit system's ability to achieve optimum allocation of water resources could be increased if its objective were stated so that the performance of the system could be measured, and if a more viable criterion were given by which alternative allocations could be ranked.

Thus far, the analysis has concentrated on an assessment of the permit system's ability to achieve optimum allocation of the state's water resources given static conditions of requirement and supply. Another important facet of the system's optimizing ability is its responsiveness to changing physical, economic, and demographic conditions. To assist in examining this aspect of the permit system, a set of terms suggested by Ciriacy-Wantrup (15) will be used. These terms, "rigidity", "protection", and "security", denote nonresponse, while the term "flexibility" denotes responsiveness (15, p. 252). With respect to the permit system, rigidity refers to the lack of permit mobility among alternative uses. Protection refers to the assurance given a permit holder by the permit system that his water right is protected against unlawful acts by others; this is a legal topic, beyond the scope of this analysis. Security can be thought of as protection against tenure uncertainty, which is the possibility that a right may be lost to superior rightholders, or physical uncertainty, which is the possibility of loss of right due to flow variability.

One component of the rigidity which the Iowa permit system possesses could decrease absolutely over time. This rigidity is found in certain rights which were to be preserved after the enactment of the statute (48, sect. 455A.23). As these prior uses are discontinued, more flexible allocations may take their place. However, the requirement that a low flow be protected in all watercourses (48, sect. 455A.22) represents a component of rigidity which could increase in relative importance in a time of general shortage. The uses for which the minimum flow is protected are the nonregulated uses, which are assured a top priority as long as there is flow in the stream. If flow decreases, regulated uses may be required to cease, while nonregulated uses are assured an increasing share of available flow.

Security is provided in the statute against both tenure uncertainty and physical uncertainty. Some protection against the physical uncertainty of variable flow is accorded to nonregulated uses by established low flow standards. Protection of this flow requires that consumptive uses cease when they endanger the protected flow, while nonconsumptive withdrawals may continue as long as flow is adequate. Thus, regulated consumptive uses are least secure, regulated nonconsumptive uses more secure, and nonregulated uses most secure from physical uncertainty.

Protection against tenure uncertainty is practically complete for nonregulated uses, as long as the minimum flow requirement stands. For permitted uses, this protection is less certain, as there are several ways a permit may be revoked or suspended. Violation of the terms of the permit or nonuse of the allocated water allow the water commissioner to revoke the permit (48, sect. 455A.20). In cases of emergency, the commissioner may suspend the permit for no more than thirty days.[6] Otherwise, permission of the user is required before a permit may be revoked (48, sect. 455A.20), and the permit is secure for its duration, at most ten years (48, sect. 455A.20).

Flexibility, reflecting responsiveness in the permit system to changing conditions, is limited. Partially because of the security aspects discussed above, allocation may be inflexible for the duration of the permits granted. Greater inflexibility arises from the stipulation that permits can be transferred only if ownership of the property on which the water is used is transferred (48, sect. 455A.30). If changes in demand or supply make current allocations suboptimal, to move toward optimization requires not only the ability to change existing allocations, but the ability to identify those uses which would increase total benefit. The Iowa water permit system possesses neither of these abilities.

Economic Interpretations of Legal and Permit System Allocation

Graphical representation of a hypothetical production function

Figure 1a shows the shape of a hypothetical production function for the jth product, embodying the assumption that marginal physical product (mpp), as shown in Figure 1b, first increases and then diminishes as water use increases, if all other inputs are held constant. Such a production function can be divided into three stages as follows (32, pp. 122-123):

Stage I: $\{x \mid 0 \leqslant x \leqslant a\}$; point a is the point of maximum average physical product;

Stage II: $\{x \mid a < x \leqslant b\}$; at point b, total product is a maximum and mpp = 0;

Stage III: $\{x \mid x > b\}$. Beyond b, total product declines as additional water is used; mpp $<$ 0.

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[6] (48, sect. 455A.20). In Minnesota, a permit may be cancelled for any reason for protection of the public interest (29, p. 241).

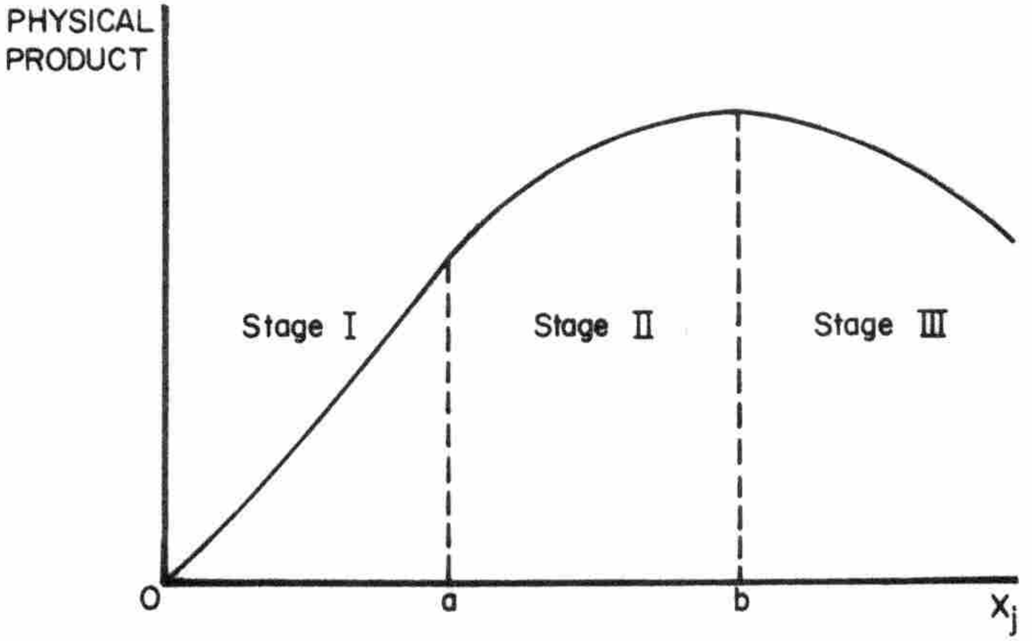

Figure Ia. A Hypothetical Production Function - Total Physical Product

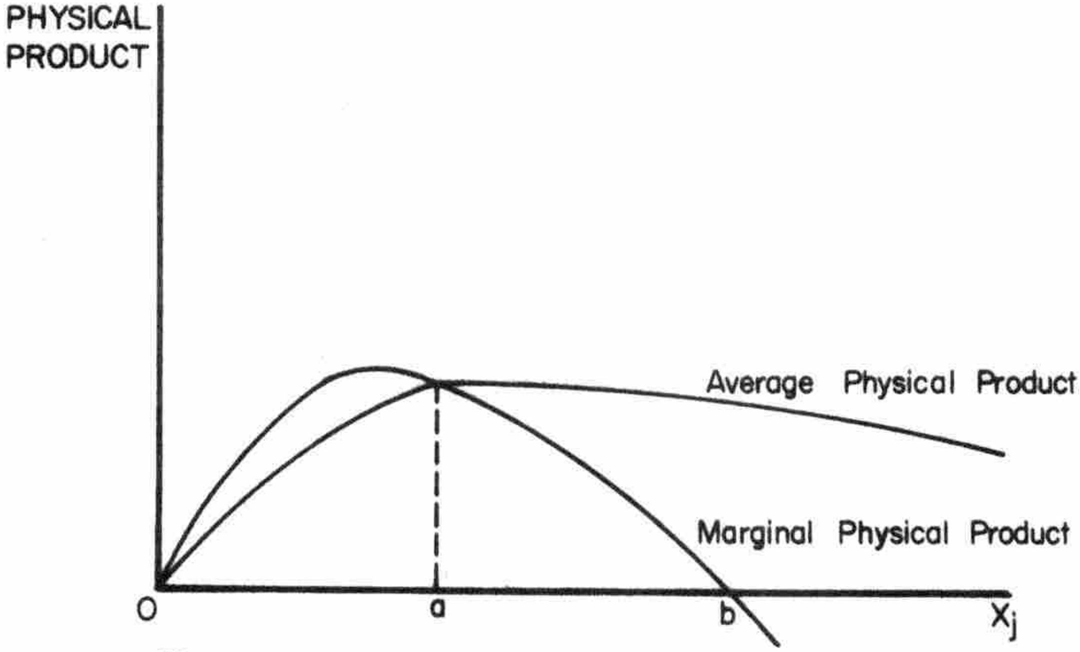

Figure Ib. A Hypothetical Production Function - Average and Marginal Physical Product

It can be shown that no rational producer would continue operating in stage I, where average physical product is increasing.[7] Instead, he would increase water use beyond point a. A rational producer, in most cases, would also restrict water use to $x \leqslant b$, for beyond b, water is not only wasted, but total physical product is decreased with every additional unit of water.

Thus, analysis can be restricted to stage II, in which a rational producer would operate, and stage III, where production implies that water is being wasted. From the general necessary conditions for optimum resource use developed in Chapter II, two conclusions are obvious. First, if total water use is less than the available amount, vmp = $\lambda = 0$,[8] and optimum allocation occurs at b. Second, if total potential water use is greater than or equal to the available amount, vmp = $\lambda > 0$, and optimum allocation occurs between a and b, with the particular allocation depending upon the value of vmp.

Recognition of necessary conditions in centralized allocation systems

In measuring relative worth of water in alternative uses, the appropriative doctrine, the riparian doctrine, and the Iowa permit system all depend upon criteria developed in statutory or case law. For all systems, the criteria are similar. The appropriative doctrine ranks uses according to their relative benefit, contingent upon the

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[7] This can be seen intuitively by considering that as use of x is increased from $x = 0$ to $x = a$, the producer experiences increasing returns to x; in this range, increasing x increases the return to all units of x. It would be logical to continue to increase x until the point $x = a$ is reached. Furthermore, in the interval $0 \leqslant x < a$, the marginal physical product of any fixed input is negative (32, p. 123).

[8] vmp = output price x mpp; under the assumption of perfect competition made in Chapter II, output price is constant regardless of the level of output. Therefore, the vmp curve and the mpp curve are similarly shaped, and are equal to zero at point b.

existence of a preference ranking within the jurisdiction. The riparian doctrine states the priority of natural uses over artificial uses, and maintains that the riparian owner has the right to reasonable use of the flow across his riparian land. In times of scarcity, allocation is made based on relative reasonableness, defined in terms of benefit to the user. The permit system in Iowa requires that a use be beneficial to the user, and as in the riparian and appropriative doctrines, disallows waste.

A use is beneficial to the user if the vmp of water in that use is positive. This fact implies that such a user will be operating in stage II of his production function, as discussed in the previous section, since no rational producer would continue operating in stage I. Water use will also not occur in stage III, since in stage III the use is both nonbeneficial and wasteful.[9] The only two points, therefore, which the appropriative doctrine, the riparian doctrine, and Iowa's permit system define are those two points where vmp is zero: at zero input use, and at maximum total physical product. No point between these two extremes is defined.

For abundant water supplies, this lack of definition is unimportant, since vmp of abundant water is zero. However, if the supply is scarce, then vmp becomes positive. Optimum allocation of a scarce supply occurs at some point where vmp is positive; none of the three legal allocation systems identifies such a point without judicial or administrative procedure.

Iowa's permit system, by inhibiting free transfer of water rights (48, sect. 455A.30), precludes the operation of a market for water rights. Such a market would, in theory, tend to allocate water rights in an optimal manner. Since no market exists, and the permit system does not define the necessary conditions for optimum resource use, the following chapters describe the construction of a model to generate estimates of optimum resource use.

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[9] This is untrue if the production function is horizontal in the interval $x > a$. In this case, water use beyond $x = a$ does not produce negative benefits, but is nevertheless wasteful.

CHAPTER FOUR: HYPOTHESIS

Hypotheses, as guides to inquiry, are propositions concerning cause-effect relationships (33, pp. 41-53). Depending upon the objectives of an analysis, its hypotheses may be described as problem delimiting, diagnostic, or remedial (86, p. 24). Problem delimiting hypotheses illustrate the nature of a problem in terms of divergences between existing situations and desired goals. Diagnostic hypotheses attempt to explain why a problem exists, and remedial hypotheses describe methods by which the desired goal can be reached.

The hypothesis constructed to guide this analysis is problem delimiting, and it is based upon the following two assumptions:

1) Iowa's water permit system was designed to be an optimizing institution, one which allocates water rights such that progress is maximized toward some goal or set of goals.

2) The set of goals toward which the permit system is designed to move includes goals of equity, security, and economic growth.

The equity concept in the permit system's assumed goal set refers to a distribution of wealth throughout society consistent with generally accepted standards of distributive justice (78, pp. 59-69). In the context of water rights allocation, the goal of security refers to the assurance of a right holder that his water right will not be lost for at least some specified time period.[1] There are static and dynamic aspects of the goal of economic growth. Investment is the dynamic aspect,

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[1] According to Ciriacy-Wantrup (15), a water right holder faces physical uncertainty, for flow may not always be sufficient to meet his needs; legal uncertainty, in that his water right may be infringed upon due to the illegal acts of others; and tenure uncertainty, whereby his water right may be lost due to the actions of others with prior or superior rights. Dams, impoundments, and other physical structures provide a degree of security against the physical uncertainty of variable supply, and legal systems provide security or recourse against loss under legal uncertainty. Security of tenure, however, is one of the protections which systems of water rights allocation seek to provide.

inasmuch as net investment in capital goods increases the productive capacity of the economy. The static aspect is efficiency, which refers to that allocation of resources and output which maximizes social benefit (78, pp. 59-69).

Given assumptions (1) and (2), it is possible to construct the following general hypothesis: Iowa's water permit system, in situations of insufficient water supply, will optimally allocate Iowa's water resources, where optimum allocation maximizes movement toward the set of goals assumed in (2). A study of this general hypothesis would require both static and dynamic analyses of the effect of water allocation on equity, security, efficiency, and investment in Iowa. Consideration of all these questions is beyond the scope of this analysis; therefore, a more restricted, operational hypothesis is used as a guide.

Only efficiency, the static dimension of economic growth, is examined in this analysis. Further, analysis will be confined to considerations of short-run efficiency.[2] In addition to defining a problem of manageable proportions, the analysis focuses on this single goal for two reasons. First, the theory of short-run efficiency is relatively complete, and several analytical techniques exist upon which a model can be built. Second, it will be shown in Chapter V that the model used in this analysis can be adapted to account for considerations of equity, security, and investment. With these restrictions, the "working hypothesis" (33, pp. 46-47) is as follows: Iowa's water permit system will achieve efficient short-run allocation of Iowa's water resources in situations of scarce water supply.

It was noted in Chapter I that since the inception of the permit system in 1957, only two permit applications have been refused, and that each of these requested a permit to dispose of excess surface water. Therefore, the hypothesis above will not be empirically tested in this

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[2] That period over which all inputs are variable is the long run (32, pp. 107-108). One of the inputs held constant in the short run is capital, which is consistent with the decision not to examine the investment goal.

study, since the problem of allocating an insufficient water supply apparently has not arisen.[3] However, this hypothesis will be illustrated by a linear programming model. This model will show optimum water allocation in a given situation, which can be compared with the allocation which might result from operation of the permit system in the same situation.

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[3] According to Hines (43, pp. 38-39, note 179), permits are often granted according to terms more restrictive than those requested by the applicant. In some cases, the amount of water granted is less than that applied for, but these reductions are often the result of an applicant's request for more than a reasonable amount of water for his use (43, p. 38). Shortages of water which necessitate critical allocation decisions apparently have yet to occur.

CHAPTER FIVE: GENERAL LINEAR PROGRAMMING MODEL FOR RESOURCE USE

A linear programming model was chosen in this analysis primarily because of the excellent correspondence between the requirements of the problem and the features of this type of model. Because of the wide acceptance and frequent use of linear programming as an empirical tool in economic analysis, no theoretical discussion of the technique will be given.[1] Instead, the points of correspondence between the problem and the tool will be summarized.

As an analytical technique, linear programming can be applied to many types of situations. A common type of problem is that of finding the optimum levels of a number of alternative activities, when these activities are constrained by fixed quantities of available inputs. This is analogous to the problem under consideration in this thesis: identification of the set of activities which makes optimum use of a fixed water supply.

Another advantage of using a linear program can be found in the primal-dual relationship.[2] The relevant implication of this relationship can be summarized in the following way. For every linear programming problem there exists simultaneously another programming problem, called the "dual" of the original problem (which is known as the primal). The primal-dual relation is symmetric, and if the primal is a maximization problem, the dual is a minimization problem. Further, if the primal objective is to maximize the value of output subject to input constraints, then the objective of the dual is to minimize the "shadow prices" (5, p. 110), or internally imputed values of the inputs. Thus, the solution to the dual generates for each input in the primal a value which corresponds to the Lagrange multiplier discussed in Chapter II.

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[1] Full discussions of linear programming can be found in (5, pp. 70-128), (36), and (82, pp. 88-171).

[2] For discussions of duality, see _supra_, note 1.

The dual optimum solution tells by how much the value of the objective function would be increased if an additional unit of each of the primal inputs were available.

A further advantage in the use of a linear programming model is its flexibility. Such a model can be used to describe allocation problems involving only a few alternative uses for a scarce water supply as well as allocation problems in which there are many diverse alternative uses. A linear program can also be used for single-period analysis or, with minor modifications, for multi-period analysis in a recursive framework. For multi-period analysis, a recursive linear program can also be linked with a simulation model which provides exogenous data to the linear program for each succeeding time period. For any period, this information is based on the reaction of simulated physical or economic systems to the results of the linear program's optimum solution in the preceding period.[3]

Model Structure

The model described in this chapter is simplistic in its nature, endeavoring to provide the desired information with a minimum amount of required data input. Only the short run, as defined in Chapter IV, is considered. However, within this limited scope, the model exhibits dynamic properties in considering seasonal variation in water requirements and supplies. The time period of the model is defined as one year, but in applying the model to any given area, the year can be partitioned into single months or groups of months. The time periods would be constructed to illustrate seasonal fluctuations in water requirements and supplies and transfer of water supply from month to month through storage facilities. In the application in this study, four groups of months are defined, as shown in Chapter VI.

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[3] For an example of such an application, see (39).

The aforementioned single-year approach notwithstanding, multi-year applications can be made with relative ease. This could be done either by solving the model once for each year in the interval considered, or by solving the model for the first and last years of the interval. In either case, changing conditions would be denoted by corresponding changes in the model's coefficients and parameters between solutions.

The form of the model is as follows:

$$\text{maximize } Z = c'X^P$$

$$\text{subject to}$$

$$A \begin{bmatrix} X^P \\ X^s \\ X^r \end{bmatrix} \leqslant b$$

$$X^P \leqslant \overline{X}^P$$

$$X^s \leqslant \overline{X}^s$$

$$X^r \geqslant \overline{X}^r$$

$$\left[X^P, X^s, X^r\right] \geqslant 0.$$

The variables have the following dimensions:

1) c is a p-vector;
2) X^P is a p-vector; X^s is an s-vector; X^r is an r-vector; $p + s + r = n$;
3) A is an m x n matrix;
4) b is an m-vector.

The primal form of the model is composed of four components: a set of activity variables, $\left[X^P, X^s, X^r\right]$; a matrix A of technical coefficients; a set of constraint parameters, $\left[b, \overline{X}^P, \overline{X}^s, \overline{X}^r\right]$; and an objective function, $c'X^P$. The dual form, which generates the shadow price of each primal input, is determined once the primal is defined. The structure of each of the model's components is dictated in part by the type of information the model is intended to provide. The model is constructed to find the optimum level of water using activities in an area. No attempt is made to specify the optimum combination of activities within each agricultural or industrial water user's operation. It is assumed that this optimization has already occurred within each firm.[4] Each of the four primal components will be described in the following section.

Activities

For any time period, the model's set of activity variables represents each actual or potential use to which water withdrawn from specific sources considered in the model can be put. These activity variables fall into three subsets which can be defined as follows:

a) X^P, the set of uses demanding water as an input to a production process. The assumption that each firm has found the optimum combination of technological alternatives for the production of each of its products implies that there is only one process per product. There will be, therefore, one X_j for each product produced in the area under consideration. The production function of each of these activities is defined by assumption to be $X_j = f_j$ (water, land, labor, capital_j), $X_j \; X^P$. Each of these inputs is subject to a constraint on the amount available per time period.

In this study, most activities in X^P are represented by aggregate sectors defined in Table 22, Appendix A. These sectors are composed of a number of manufacturing or non-manufacturing activities producing the same type of output. The output of any of these sectors is a fictional product type, so that the shadow price of water in that activity does not refer to any particular product, but to an aggregate of the products in the sector. Such general shadow prices are useful in determining the value of water in these sectors, but for some activities, more specific information may be desirable.

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[4] This two-stage decision-making process is developed in (68).

Where information is desired with respect to a specific product, an activity is defined which represents the production of that product.

Meat packing, cattle feed lots, corn production, and soybean production are represented specifically in application of the model. The level of each of these activities is measured in physical output units. All other activities in X^P are represented by aggregate sectors. Output in each of these activities is measured in money valued units, which are defined below in the section discussing the model's objective function.

b) X^s, the set of uses which represent treatment of water to change its time, quality, or location characteristics. Included in this subset are municipal water treatment and water pollution control plants, as well as storage and transport facilities. Each of these activities is assumed to have a production function of the same type shown in (a) above for X^P, the producing activities. The unit in which activities in X^s are expressed is one thousand gallons of water.

c) X^r, in part, a set of public water uses in which water can be conceived as a commodity, yielding utility directly by its use. Residential use and recreation are included in this subset, as well as an activity for each surface stream source in the model, representing use of water to satisfy the "protected low flow" requirement of the permit system (48, sect. 455A.1). Also included in X^r is an activity to represent the amount of water which must remain in a source to service the rights of downstream permit holders who are not explicitly represented in the model. Activities in X^r are measured in units of one thousand gallons of water.

Technical coefficients

The model's matrix of technical coefficients consists of ratios defining the amount of each resource required for the production of a unit of each activity. For each $X_j = f_j$(water, land, labor, capital) $\forall X_j \in X^P, X^s$, there is a technical coefficient for each input in the production function. For domestic use there are two coefficients, one representing transfer of water either from a source or a treatment activity to a public use, the other representing transfer of waste water from domestic use to a treatment facility. For recreation and flow protection, a single coefficient for each represents net use per period from sources in the model.

All water use coefficients show net consumption per unit of activity, except where withdrawals are from one source and discharge is into another source, or where water inputs and waste water outputs are treated by separate facilities. In each case, both coefficients must be explicitly accounted for to show movement of water from one supply to another. Consumption of location, quality, or time utility in water supplies can be illustrated in this way, differentiating water supplies according to these three parameters.

Constraint parameters

The set of constraint parameters contains components representing the maximum amount of each resource available to the model per time period. Water, land, and labor are represented by elements of the b vector.[5] Each of these resource classes is heterogeneous and can be divided into more homogeneous subclasses. The number of

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[5] Within the system $Ax \leq b$, the following specific constraint inequalities can be identified:

let $\sum_{j=1}^{n} a_{1j}\, X_j \leq b_1$

$$\sum_{j=1}^{n} a_{2j}\, X_j \leq b_2$$

$$\begin{matrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{matrix}$$

(footnote continued on following page).

subclasses used is determined by the amount of detailed information desired from the model. A shadow price is generated for each resource subclass delineated, but more detailed input data are required as the number of subclasses increases.

Consistent with this relation between input data and output information, the water and land resources in the model are differentiated, while the labor resource is considered homogeneous. This is done for the water resource because information is desired concerning the differential value in use of various water supplies. Land is subclassified because there is evidence[6] that some agricultural activities have a

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(footnote continued from previous page)

$$\overset{n}{\underset{j=1}{}} \quad a_{uj}\, X_j \quad b_u \text{ express the area water use constraints;}$$

$$\overset{n}{\underset{j=1}{}} \quad a_{u+1j}\, X_j \quad b_{u+1}$$

$$\overset{n}{\underset{j=1}{}} \quad a_{u+2j}\, X_j \quad b_{u+2}$$

$$\begin{matrix} \vdots & & \vdots \end{matrix}$$

$$\overset{n}{\underset{j=1}{}} \quad a_{m-1j}\, X_j \quad b_{m-1} \text{ express the area land use constraints; and}$$

$$\overset{n}{\underset{j=1}{}} \quad a_{mj}\, X_j \quad b_m \text{ express the area labor force constraint.}$$

[6] In Arizona, where water is generally scarce, Young and Martin (107) showed personal income generated per acre foot of water used to be approximately 1000 times higher in manufacturing than in the highest-valued crop use.

significantly lower return to a scarce water resource than some industrial activities. By considering return to water used in alternative activities on different types of land, planning decisions could be made which would enhance the movement of water used on low productivity agricultural land into higher productivity agricultural or industrial uses.

Labor supply could also be considered in this manner. However, labor is a relatively mobile resource both geographically and occupationally. Further, knowledge of the magnitude of the return to water used by labor subclasses is of doubtful value. For these seasons, labor is treated as a homogenous resource class.

Available capital inputs to activities in the model are considered to be fixed in any time period. Each activity which requires capital operates under a constraint on the available amount of fixed plant and equipment. This constraint can be expressed either as a physical quantity representing the production capacity of each of the activities, or as the dollar value of available fixed plant and equipment. For activities in the producing sector, X^P, the constraint is denoted by $\overline{X}^P$; for water supply activities, X^s, the constraint is denoted by $\overline{X}^s$. By the hypothesis in Chapter IV, investment is disregarded in this analysis, but $\overline{X}^P$ and $\overline{X}^s$ could be changed between periods in a dynamic analysis to allow for consideration of investment.

Objective function

The model's objective function follows from the hypothesis in Chapter IV, in which the analysis is restricted to considerations of short-run efficiency in water allocation. By definition (78, p. 148), efficient allocation is that which maximizes social benefit gained in the use of water; therefore, the model's objective is to maximize social benefit.

Social benefit is difficult to measure, for it includes not only the dollar-valued output of goods and services, but also many items which have no readily discernible market value. For instance, social benefit from water use includes the benefit derived from such water uses as recreation and conservation. Further, water is necessary to sustain life, and its value in fulfilling this function is difficult to quantify. Because of these difficulties, it is necessary to find a proxy for social benefit.

In this study, the proxy used is based upon the value of output of the producing activities in the model; these activities are represented by the elements of X^P. Value of output is represented by $\sum_{j=1}^{p} P_j X_j$, where P_j is the unit price of the output of X_j. However, value of output may include payments to factors not located in the area affected by the hydrologic system under consideration. Therefore, for each activity X_j, these payments are excluded from the objective function by the method described below.

For each activity, total value of product is assumed to be exhausted by payments to the various inputs and factors of production, as expressed by the following relationship:

Product value/unit of X_j = Wages and Salaries/unit of X_j + Materials Cost/unit of X_j + Other Income/unit of X_j.

The several terms of this relationship are defined as follows:

a) Wages and Salaries/unit of X_j is the portion of product value paid to those whose labor is expended in production of the jth product.

b) Materials Cost/unit of X_j is the portion of product value paid to purchase the materials which are part of the jth product, including materials imported from outside the area.

c) Other income/unit of X_j is the remainder of product value, including profits, return on capital invested, and rents according to land and water in the production of X_j.

The element in the objective function corresponding to X_j is

c_j = Product Value/unit of X_j-Materials Cost/unit of X_j = Wages and Salaries/unit of X_j + Other Income/unit of X_j.

Thus, each activity is weighted according to the portion of its product value earned by those factors of production local to the hydrologic system under study. These factors are labor, land, water, fixed capital, and managerial ability.

By excluding the cost of materials from the objective function coefficient, two problems are avoided. First, any payments for materials produced outside the model area are excluded; these payments do not represent benefit to individuals in the model area. Second, excluding materials cost insures that only the value of final production is measured.[7]

There are at least three theoretical difficulties in using a portion of product value to approximate social benefit. First, if there are any external effects present in the model area, marginal private cost and benefit may not be equal to marginal social cost and benefit, respectively. In this case, the market value of output does not represent its value to society; if marginal social benefit exceeds marginal private benefit, output value understates social benefit. If marginal private benefit exceeds marginal social benefit, which is the case where air and water pollution result from production, the value of output overstates social benefit.

Another difficulty is that an increase in output, while increasing some individual's benefit, may decrease the benefit derived by others. If this occurs, it is not possible to specify whether social benefit has increased, because no method for making interpersonal utility comparisons exists at this time (78, p. 64). A third difficulty in the use of a portion of total output value to approximate social benefit is that this measurement conceals any changes in either the quality of the several outputs produced, or in the relative proportions in which these outputs are produced (output mix). Changes in both output quality and output mix can influence social benefit.

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[7] Bread is a final product, the value of which includes the value of the flour used in its production. Flour is an intermediate product, and to add the value of bread production and the value of flour production would be to count the value of flour twice. See (75, pp. 183-186).

Notwithstanding the existence of these difficulties, the objective function of the model is defined as

$$Z = c'X^P$$

where c_j = Product Value/unit of X_j-Materials Cost/unit of X_j = Wages and Salaries/unit of X_j + Other Income/unit of X_j, $j=1,\ldots,p$. Those activities which are treated specifically are measured in physical production units; the corresponding element in c is the price per unit for that product. Those activities represented by aggregate sectors are measured in dollar-valued output units. These units are defined to be the amount of production required to generate a one dollar increase in product value from that sector. c_j, as defined above, is the product value per unit of output. Therefore, the unit of measurement of each activity in X^P represented by an aggregate sector is c_jX_j, and the coefficient in the objective function is actually unity. The objective function of the model is identical to that defined above, $Z = c'X^P$.

The coefficients in the objective function associated with elements in X^s, the water-supply activities, and X^r, the residual social water uses, are defined to be zero in all activities except one because most water in Iowa presently has no market price. Therefore, the value of a unit of treated water, which derives from the use of that unit in production of consumption uses, cannot be directly measured, nor can the value of a unit of water used in human consumption, recreation, or low-flow protection. The activities in X^r are instead constrained to appear in the solution, while the activities in X^s will appear at some positive level due to linkages with activities in both X^P and X^r.

In one application of the model, one activity in X^s is assigned a negative coefficient in the objective function. This activity represents treatment of a polluted water supply. It differs from the activity representing treatment of a less polluted supply only in the objective function coefficient assigned to each. The negative c_j is the amount per unit by which treatment costs are increased by the presence of pollutants in the water supply.

Interpretation of the Solution

Solution of the model yields an activity set $\left[X^{P*}, X^{s*}, X^{r*}\right]$ which maximizes the value of the objective function. Optimal water use in activities in X^P must be calculated indirectly from the optimal solution since these activities are expressed in terms of output units. For X_j*, $\forall X_j \in X^P$, water use is calculated by multiplying X_j*, the optimum level of that activity, by its technical coefficient of water use (water used per unit of output). From any water supply represented by b_α, the water used in activities in X^P would be equal to $\sum_{j=1}^{p} a_{\alpha j} X_j*$. Total water use in activities in X^P from all sources would be obtained by $\sum_{i=1}^{u} \sum_{j=1}^{p} a_{ij} X_j*$ for all water sources, $b_1, b_2, \ldots b_u$.

Activities in X^s and X^r are expressed in units of one thousand gallons of water. Total water use in these activities would be $\sum_{i=1}^{u} \sum_{j=p+1}^{n} a_{ij} X_j*$, while total water use in all activities would be given by $\sum_{i=1}^{u} \sum_{j=1}^{n} a_{ij} X_j*$.

Also generated are shadow prices for each constraint. For the b vector, the shadow prices represent the value of an additional unit of land, labor, or water resource. For $\overline{X}^P$, $\overline{X}^s$, and $\overline{X}^r$, the shadow prices have an analogous interpretation. However, the shadow prices associated with each parameter in $\overline{X}^r$ represent the amount by which the objective function would be increased if minimum requirements for public use were lowered by one unit. This value, while not a price, is an opportunity cost measure which could be an aid in planning, expecially with respect to water reserved for residential, recreational, or flow protection uses.

Changes in Parameters

To express actual or proposed changes in water supply or requirements in an area, the parameters of the model can be changed. Population growth, for example, can be expressed by increasing the amount of water reserved for residual and municipal use. Increased recreation use can be represented analogously. Increases in industrial or agricultural requirements can be shown by raising the limit of the constraining resource (land, labor, capital, or water) and allowing that activity to expand.

Secular changes in water quality levels can also be represented by shifting quantities of water from high quality supplies in the model to lower quality supplies. If an increased cost is shown to be associated with use of lower quality supplies, the value of treatment will be reflected. By identifying the source of quality degradation and identifying the consequence area where treatment finally takes place, the model may also provide planning information for quality improvement programs.

Adapting the Model to Multi-Dimensional Goals

Considering only the one-dimensional goal of short-run efficiency implies that this goal is independent of the goals of equity, security, and investment. If the four goals are independent, maximization with respect to any single goal is not inconsistent with maximization in terms of any or all of the other three. Operationally, this assumption may be unwarranted. Security and investment are related in that, for planning purposes, the length of time over which a water-related investment must pay for itself (the planning horizon) depends upon how long the investor's water right is assured. For a given rate of return, the maximum feasible investment decreases as the planning horizon becomes shorter, while for investments of a given size, higher rates of return must be forthcoming as the planning horizon is shortened. Equity and efficiency are also not mutually exclusive. Efficient resource allocation given current income distribution may be inefficient if income is redistributed to be more equitable.

In a model designed to show short-run efficiency, these interactions can be shown as additional constraints. As an example, if an increase in investment is desired, those activities in which investment is desired can be granted permits for the maximum allowable period, shown in the model by reducing the amount of available water in succeeding time periods by this secured amount. The model also shows which activity will have the greatest direct increase in output and employment from the use of additional water.

Investigation of the effect of permit security could take the form of a constraint representing, in successive iterations of the model, the amount of water secured from reallocation by permits in force. Under each different assumed permit duration, the model could be reiterated yearly for a specified time period, and the present value of each year's production computed. These present values could then be summed over the time period for which the model was run, and compared with the present values associated with each assumed permit duration.

CHAPTER SIX: APPLICATION OF THE GENERAL MODEL

The model described in Chapter V is applied to two situations in this chapter. One application is to a hypothetical water use situation. Apparently, no water shortage exists presently in Iowa which is serious enough to affect a diverse group of economic activities.[1] Therefore, the hypothetical situation is constructed to illustrate the use of the model more completely than could application to an existing situation of limited scope in Iowa.

The second application of the model is to the use, by existing activities, of water from a shallow sand and gravel aquifer, located near an Iowa town of approximately 5000 population. This application encompasses fewer activities and a more abundant water supply than the hypothetical situation, but it illustrates what could be considered a typical application of the model in a real situation. These applications, designated I and II respectively, are described in the following sections.

Application I: A Hypothetical Water Use Situation

The water use situation under study in application I is illustrated in Figure 2. In this situation, water may be used in crop and livestock agriculture, industry, and domestic uses. The water supply on which these uses depend is a stream with a 20 square mile drainage area. The flow in this situation is assumed to be dependent upon rainfall in the basin. The mathematical relation expressing this dependence is shown in a later section discussing the model's resource parameters.

The annual time frame of the general model described in Chapter V is partitioned into four time periods in application I. These time periods are constructed to reflect the seasonal variation in water supply and in the water requirements of the agricultural activities. The time periods are as follows:

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[1] Interviews with officials of the Iowa Water Commissioner's staff and the U. S. Geological Service, as well as with members of the Iowa State University faculty in several departments failed to reveal any situation of scarcity.

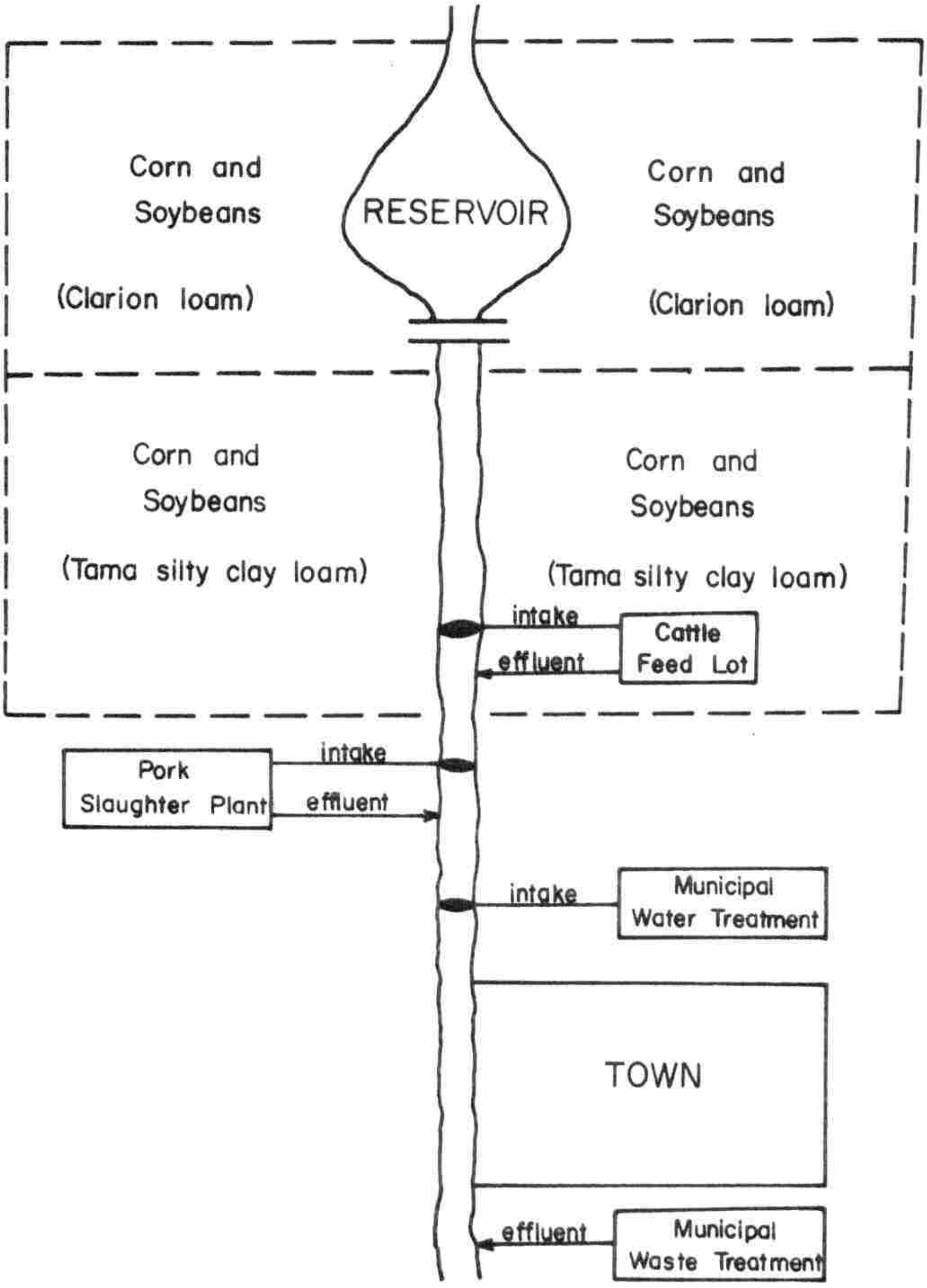

Figure 2. Spatial Arrangement of Activities in Application I

period 1 - November through April;
period 2 - May and June;
period 3 - July and August;
period 4 - September and October.

Figure 3 shows these time periods superimposed upon the distribution of annual rainfall by months. It can be seen that period I contains the winter months of low rainfall while period 2 contains the spring months, which have the highest average rainfall of the year. Period 3 contains those months during which rainfall reaches its lowest level for the summer season, while period 4 contains a peak which occurs as rainfall increases from the low level of period 3 and begins to decrease to the winter season low rainfalls.

Crop water requirements during the growing season also fluctuate. The time periods defined above serve to isolate the period of maximum crop water requirements for crops considered in application I. According to Shaw, et al. (80a), estimated average water requirements for corn during the periods defined above are as follows:

period 1 - 4.9 inches;
period 2 - 7.1 inches;
period 3 - 10.7 inches;
period 4 - 5.1 inches.

The time periods of the model, as they are defined, allow for the juxtaposition of rainfall and crop water requirements in such a way as to isolate those periods during which supplemental irrigation may be required.

Activities

Figure 2 shows the arrangement of activities in the situation represented in model application I. One central feature of this arrangement is the relationship of the pork slaughtering activity to the town. If, as is assumed in the model, significant pollution results from slaughter operations, the town downstream, which has no alternative supply, must bear the cost of removing this pollution from the water withdrawn for municipal use. This assumed relationship creates a

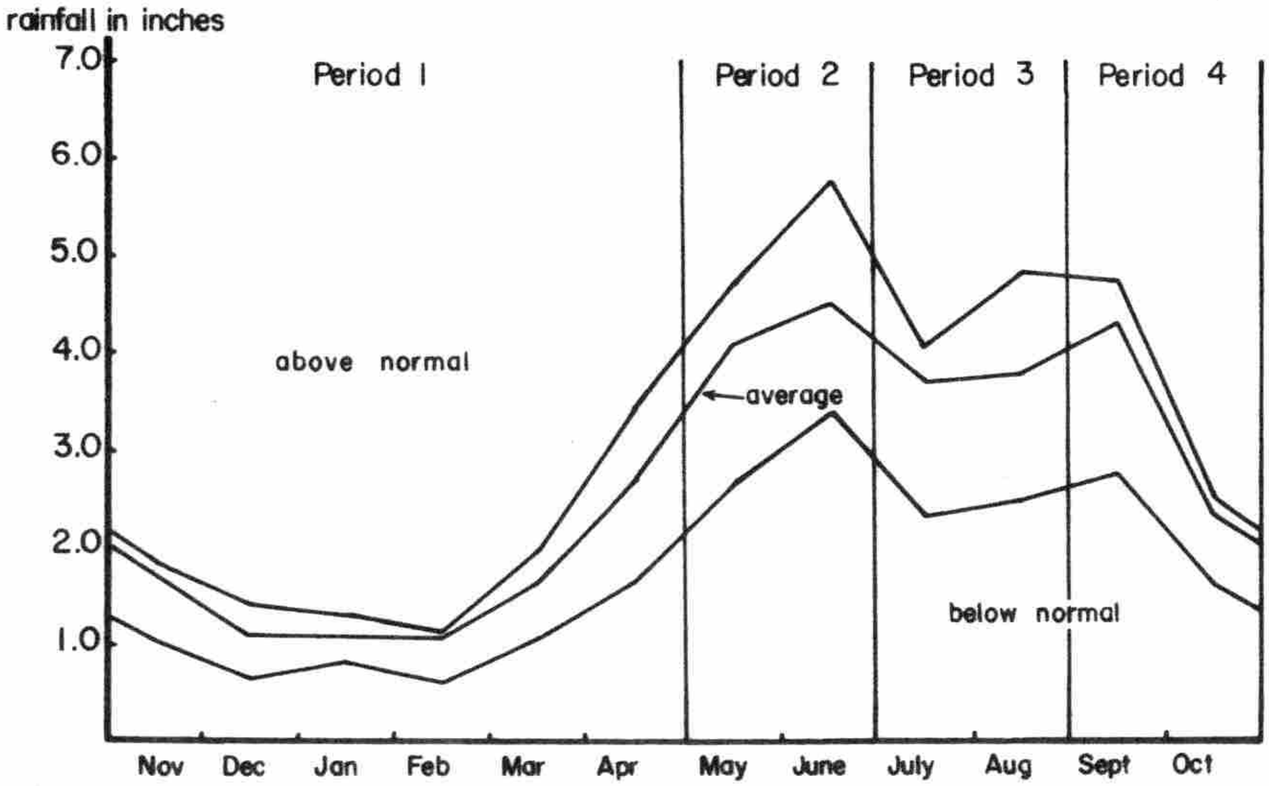

Figure 3. Distribution of Rainfall Through Assumed Time Periods, Application I(80, p. 7)

framework in which to study the feasibility of using the stream between the pork plant and the town as an effluent carrier.

The reservoir is included to illustrate the value of transfer of water between time periods. Inclusion of this facility also allows consideration of the competition between recreation, for which a stable reservoir level is desirable, and water storage, for which a fluctuating water level may at times be necessary.

Two soil types are shown in the hypothetical situation so that comparisons can be made between the value of water used in crop production on each soil type. Further, the cattle feed lot operation is located on irrigable land, so that the value of water in different agricultural uses on the same soil type can be calculated.

In this section, each activity in application I is defined. Each activity's coefficient of resource use and its coefficient in the objective function are shown in tabular form. The derivation of each of these coefficients is explained in the text. The four variables listed below for each activity designate that activity in each of the four time periods of the model.

The producing sector (X^P)

Agricultural activities There are five activities in the agricultural portion of the producing sector. They are a cattle feed lot, corn production on the two soil types, and soybean production on the two soil types. Aside from differences in yield due to soil type as shown in Table 19, Appendix A, the crop activities are defined similarly, so that comparisons can be made of the value of water in each crop on the same soil type.

Crop activities on Tama silty clay loam are designated Corn Production I and Soybean Production I. Crop activities on Clarion loam are designated Corn Production II and Soybean Production II. Each agricultural activity withdraws water from the stream shown in Figure 2. The activities are defined as follows:

X_1, X_{25}, X_{49}, X_{73} - Cattle Feed Lot, which utilizes land, labor, water withdrawn from the stream in Figure 2, and capital as illustrated

Tables 1a and 1b. The activity involves feeding heifer calves, which are purchased at 400 pounds, fed a high roughage feed for 288 days, and sold at 925 pounds. Feed lots can be considered completely consumptive water uses, since liquid waste from the cattle is the only discharge of intake water. Most of this waste would either evaporate or infiltrate into the soil, never reaching the stream.[2] For this reason, no discharge is shown for this activity.

Cattle feed lot operations which have no waste treatment facilities may also be significant contributors to agricultural pollution (59a, p. 1582). If feed lot solid wastes are not constantly treated, but are allowed to accumulate, these wastes may contribute only intermittently to high pollution loads in surface streams. Rainfall of sufficient intensity must occur to cause solid wastes to dissolve and run into the stream (59a. p. 1951). Such a phenomenon is stochastic, and is not considered in this thesis.

Production coefficients for labor, land, and water use in the cattle feed lot activity are based upon data given in James (51). Capital required per calf fed is based upon data in a study of feed lot operations in Northeast Iowa by Gross (35).

The relationship used to estimate revenue per unit of output is the same for each of the activities in the producing sector. This relationship, given previously in Chapter V, is

$$c_j = \text{Product Value/unit of } X_j - \text{Materials Cost/unit of } X_j = \text{wages and Salaries/unit of } X_j + \text{Other Income/unit of } X_j.^3$$

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[2] J. R. Miner, Agricultural Engineering Department, Iowa State University, Ames, Iowa. Data on the nature of cattle feed lot runoff. Private communication. July 1, 1969.

[3] Included in Other Income are returns to all non-labor factors of production, such as land, capital and entrepreneurial ability. The value of c_j differs from total product value only by the cost of primary inputs; with this single restriction, c_j may be treated as revenue.

For each activity, estimated materials cost per unit of output is deducted from estimated product value per unit of output. For X_1, Cattle Feed Lot, product value was estimated according to 1967 average prices for good and choice heifer calves to be $23.98 per hundredweight (28, Table 155, p. 107), or $221.82 for a 925 pound animal. The same calf, purchased at 1967 average prices, cost $28.00 per hundredweight (28, Table 160 - 160L, p. 113), or $112.00 for a 400 pound calf. Feeding costs are estimated by James (51, Table 2.12, p. 55) to be $73.57 for a 525 pound gain. Net revenue is as follows:

Product Value		$221.82
less Materials cost:		
calf	$112.00	
feed	73.57	
		185.57
net revenue (C_1)		$ 36.25

X_2, X_{261}, X_{50}, X_{74} - Corn Production I, the production of corn on Tama silty clay loam under high fertilization.

X_3, X_{27}, X_{51}, X_{75} - Corn Production II, the production of corn on Clarion loam under high fertilization.

X_4, X_{28}, X_{52}, X_{76} - Soybean Production I, growing soybeans on Tama silty clay loam under high fertilization.

X_5, X_{29}, X_{53}, X_{77}, - Soybean Production II, soybeans grown on Clarion loam under high fertilization.

Production coefficients for all four crop activities are shown in Table 1a and Table 1b. The data from which labor, land and capital use coefficients were compiled are contained in James (51). Coefficients of water use per unit of corn output are based on data given in Shaw, et al. (80a). These water requirements express the amount that must be withdrawn from the stream to supplement insufficient rainfall. Negative irrigation requirements, which imply an abundance of rainfall, are considered to be zero in the model. Irrigation requirements for corn and soybeans and the method of derivation of these requirements for three levels of rainfall are shown in Table 17, Appendix A.

Water requirements for soybeans are assumed to be approximately the same as requirements for corn.[4] The data given by Shaw, et al. (80a) shows consumptive crop use; return flow is assumed to be zero in each crop activity.

Product value is assumed to be the same for a crop regardless of the soil type on which the crop was grown. James (51, Table 6.8 p. 169) gives the average 1967 price per bushel for corn and soybeans, as well as estimates of the variable costs per acre (materials cost) for each crop (51, Table 8.1, p. 214). Variable costs per acre were converted to variable cost per bushel by the following manipulation:

variable cost/acre ÷ bushels/acre = variable cost/bushel.

The net revenue per bushel for each crop is as follows:

Corn (per bushel) -	
Product Value	\$1.13
less Materials Cost:	-0.55
net revenue (C_2,C_3)	\$0.58

Soybeans (per bushel) -	
Product Value	\$2.60
less Materials Cost:	-1.07
net revenue (C_4,C_5)	\$1.53

Non-agricultural activities There are eleven non-agricultural production activities in each time period in application I of the general model. Of the eleven activities, two are represented as producing a specific product, while nine activities are represented as aggregate sectors. As discussed in Chapter V, one distinction between these two methods of representation is that resource requirements in specific product activities are expressed as resources used per physical unit of output, while resource requirements in sector activities are expressed as resources used per \$1000 of product value. By assumption, none of the eleven activities has any seasonal variation in resource requirements. However, each of these activities must appear in every time period in the model. Table 2 shows, for any one of the model's

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[4] R. L. Shaw, Agronomy Department, Iowa State University, Ames, Iowa. Data on crop water requirements. Private communication. June 27, 1969.

time periods, the resource requirements of each non-agricultural producing activity. These activities are defined as follows:

X_6, X_{30}, X_{54}, X_{78} - Pork Slaughter I, which is a pork slaughtering plant operating at a rate of 230 carcasses per hour. Plant wastes from this activity are discharged into the stream after treatment for removal of biochemical oxygen demand (BOD).[5] It is assumed that the waste treatment facility of the pork plant provides adequate treatment of plant wastes at the rate of operation specified for Pork Slaughter I. By assumption, treated effluent from Pork Slaughter I does not decrease the stream's quality to such a level that existing treatment facilities of downstream users are inadequate. In terms of stream quality, the relationship between Pork Slaughter I and downstream activities is neutral. A competitive relationship exists only with respect to quantity consumption; therefore, only the amount of water consumed per unit of output in Pork Slaughter I is shown in Table 2.

X_7, X_{31}, X_{55}, X_{79} - Pork Slaughter II, which differs from Pork Slaughter I in two respects. First, the slaughtering plant is operating at a rate of 310 carcasses per hour, so that its rates of resource use differ from those of Pork Slaughter I. Second, it is assumed that, in increasing plant output, this activity exceeds the design capacity of its waste treatment facility,[6] thereby reducing the facility's efficiency and increasing the level of BOD in plant discharge. The resulting pollution load is hypothesized to be sufficient to force the town downstream from

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[5] The bacterial decomposition of organic waste in effluent water consumes dissolved oxygen. Biochemical oxygen demand (BOD) expresses the amount of dissolved oxygen which will be consumed in the decomposition of a given quantity of organic waste (56, p. 548).

[6] The treatment facility's design capacity may be exceeded because the slaughter rate has increased, because the amount of waste discharged per carcass has increased, or because poor maintenance of the treatment facility has reduced its capacity.

Table 1a. Seasonal resource requirements per unit of output in agricultural activities, application I

Resource	Period 1				
	X_1 Cattle feed lot (per head)	X_2 Corn I (per 100 bu.)	X_3 Corn II (per 100 bu.)	X_4 Soybeans I (per 100 bu.)	X_5 Soybeans II (per 100 bu.)
Land I (acres)	0.004^a	0.0^b	-	0.0^b	-
Land II (acres)	-	-	0.0^b	-	0.0^b
Labor (workers)	0.60^c	0.18^c	0.18^c	0.25^c	0.29^c
Water (gallons)					
below normal rainfall	$1,810.0^d$	0.0^e	0.0^e	0.0^e	0.0^e
normal rainfall		0.0	0.0	0.0	0.0
above normal rainfall		0.0	0.0	0.0	0.0

[a] (51, Table 5.3, p. 146)

[b] (51, Table 1.10, p. 15)

[c] (51, Table 3.1, p. 93)

[d] (51, Table 5.4, p. 147)

[e] (80a, Table 11, p. 237)

Table 1a. (Continued)

Resource	Period 2 X_{25} Cattle feed lot (per head)	X_{26} Corn I (per 100 bu.)	X_{27} Corn II (per 100 bu.)	X_{28} Soybeans I (per 100 bu.)	X_{29} Soybeans II (per 100 bu.)
Land I (acres)	0.004^a	1.02^b	-	2.9^b	-
Land II (acres)	-	-	1.1^b	-	3.4^b
Labor (workers)	0.28^c	0.20^c	0.23^c	0.66^c	0.78^c
Water (gallons)					
below normal rainfall	0.0^d	29.0^e	31.6^e	83.7^e	98.1^e
normal rainfall		0.0	0.0	0.0	0.0
above normal rainfall		0.0	0.0	0.0	0.0

Table 1a. (Continued

Resource	Period 3				
	X_{49} Cattle feed lot (per head)	X_{50} Corn I (per 100 bu.)	X_{51} Corn II (per 100 bu.)	X_{52} Soybeans I (per 100 bu.)	X_{53} Soybeans II (per 100 bu.)
Land I (acres	0.004[a]	1.02[b]	-	2.9[b]	-
Land II (acres)	-	-	1.1[b]	-	3.4[b]
Labor (workers)	0.25[c]	0.06[c]	0.06[c]	0.25[c]	0.29[c]
Water (gallons)					
below normal rainfall	0.0[d]	161.5[e]	175.9[e]	465.5[e]	545.8[e]
normal rainfall		90.1	98.1	259.6	304.4
above normal rainfall		50.9	55.4	146.7	172.0

Table 1a. (Continued)

Resource	Period 4				
	X_{73} Cattle feed lot (per head)	X_{74} Corn I (per 100 bu.)	X_{75} Corn II (per 100 bu.)	X_{76} Soybeans I (per 100 bu.)	X_{77} Soybeans II (per 100 bu.)
Land I (acres)	0.004^a	1.02^b	-	2.9^b	-
Land II (acres)	-	-	1.1^b	-	3.4^b
Labor (workers)	0.12^c	0.11^c	0.12^c	0.50^c	0.58^c
Water (gallons)					
below normal rainfall	300.0^d	17.6^e	19.1^e	50.6^e	59.3^e
normal rainfall		0.0	0.0	0.0	0.0
above normal rainfall		0.0	0.0	0.0	0.0

Table 1b. Capital requirements and net revenue per unit of output for any period in agricultural activities, application I

Resource	$X_1, X_{25}, X_{49}, X_{73}$ Cattle feed lot (per head)	$X_2, X_{26}, X_{50}, X_{74}$ Corn I (per 100 bu.)	$X_3, X_{27}, X_{51}, X_{75}$ Corn II (per 100 bu.)	$X_4, X_{28}, X_{52}, X_{76}$ Soybeans I (per 100 bu.)	$X_5, X_{29}, X_{53}, X_{77}$ Soybeans II (per 100 bu.)
Capital (dollars)	33.20[a]	45.41[b]	48.55[b]	113.77[c]	132.57[c]
Net revenue (dollars)	36.25[d]	58.00[d]	58.00[d]	153.00[d]	153.00[d]

[a] (35, Table 9, p. 47)

[b] (51, Table 8.1, pp. 213-214)

[c] (51, Table 8.2, pp. 215-516)

[d] See text for sources and derivation.

the slaughter plant to incur higher treatment costs. For the purposes of this model, it is necessary only to specify by what amount downstream treatment costs are increased, and not to specify the nature of the pollution load which causes the increase. The specific cost increase will be discussed in a later section describing the town's water treatment facilities.

As shown in Figure 2, water is withdrawn from and returned to the stream by both pork slaughter activities. Pork Slaughter II, if it enters the solution of the model, however, adds significant pollution to the stream, consuming entirely the supply of relatively unpolluted water designated in the model as stream I. At the same time, Pork Slaughter II creates a new water supply with lower quality characteristics, designated stream II. Defining two separate supplies in this way emphasizes the artificial scarcity of stream I water created by the inefficient waste treatment of Pork Slaughter II. The model generates the shadow price of any scarce resource, so that the cost of using the stream as an effluent carrier between the pork plant and the municipal water treatment facility can be determined.

Labor use and capacity coefficients for Pork Slaughter I and Pork Slaugher II are calculated from data given in Daellenbach (21). Water use coefficients are based on a survey and analysis of five meat packing operations by Thorton and Frederick (84a) and also on data gained in personal interview with production personnel at selected meat packing plants in Iowa.

New revenue is the same in both Pork Slaughter I and Pork Slaughter II. Daellenbach (21, Appendix 2, p. 131) shows identical Material Costs for 230 carcasses per hour and 310 carcasses per hour. Using the 1967 average wholesale value, carcass and by-products (28, Table 203A, p. 140), as Product Value, and including in Materials Costs the average 1967 price for 200-220 pound barrows and gilts (28, Table 203A, p. 140), net revenue is as follows:

Pork Slaughter (per carcass)	
Product Value	\$59.62
less Materials Costs	
(assuming 210 pount carcass)	46.49
net revenue (C_6,C_7)	\$13.13

The remaining activities in the producing sector of the model are represented by aggregate industrial and trade sectors. Table 18, Appendix A lists thirteen such aggregate sectors which encompass economic activity in agriculture, manufacturing, trade, and service industries. Each of these sectors represents a number of individual industries, each producing a similar product. The industries included in each sector are denoted by the Standard Industrial Classification industry code numbers corresponding to that sector. The sectors defined in Table 18 may be used as activities in the model wherever information concerning water use in the production of specific products is not desired.

The coefficients of resource use for each of the thirteen sectors are given in Table 2. These coefficients are based on data given in Barnard (3,4) and McMillan (66).[7] The general method of calculation for these coefficients is described below.

Data given in the sources cited above express, for each sector, capital, water, and labor requirements per unit of gross output. Capital required per dollar of gross output (3, Table 8, p. 53) and water intake and discharge per dollar of gross output (4, Table 4, p. 14) are given directly. The labor requirements data (66, Table 29, p. 127) are given in terms of dollars of gross output per worker; the reciprocal of this ratio is the workers required per dollar of gross output.

This revised labor coefficient, the capital coefficient, and the water coefficients must be adjusted to find the amount of each resource

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[7] Barnard defines fourteen sectors in his work (3,4). McMillan (66) defines only thirteen; the transportation and the communication and utilities sector are combined into one, entitled Regulated Industries. In aggregating the resource use coefficients given by Barnard for the two separate sectors, each coefficient was weighted by the proportion of that sector's output to total output in both sectors.

required per unit of value added in that sector in the area under study. In Chapter V, the coefficient in the objective function for any activity X_j was defined to be

c_j = Product Value/unit of X_j - Materials Cost/unit of X_j.

Therefore, that portion of gross output value in a sector which is used to purchase materials for the production of that sector's output must be calculated and deducted from gross output value.

For any one of the thirteen sectors listed in Table 18, Appendix A, the sum of that sector's purchases, per dollar of gross output, of intermediate goods from the other twelve sectors and from states outside Iowa (Table 19, Appendix A) represents that sector's materials cost per dollar of output. Gross output minus materials cost is value added, the portion of output value which is earned by labor, management, and capital factors of production. Thus, for the jth sector, the resource coefficients are computed as follows:

$$\text{Labor/dollar value added in } X_j = \frac{1}{\frac{\text{Output}_j}{\text{Worker}_j}} \cdot \frac{1}{k_j}$$

where output per worker is given by MacMillan (66, Table 29, p. 127), and k_j = value added per dollar of gross output.

$$\text{Capital/dollar value added in } X_j = \frac{\text{Capital}_j}{\text{Output}_j} \cdot \frac{1}{k_j}\ ;$$

the capital output ratio is given in Barnard (3, Table 8, p. 53).

$$\text{Water intake/dollar value added in } X_j = \frac{\text{Water intake}_j}{\text{Output}_j} \cdot \frac{1}{k_j}\ ;$$

$$\text{Water discharge/dollar value added in } X_j = \frac{\text{Water discharge}_j}{\text{Output}_j} \cdot \frac{1}{k_j}\ ;$$

Water intake and water discharge per dollar of gross output are given in Barnard (4, Table 4, p 14). Since each unit of output in a sector activity generates $1000 value added, the coefficient in the objective function is $1000 for any sector activity.

In application I, activities X_8 through X_{17} in period 1, and the corresponding activities in periods 2, 3, and 4 are represented by sectors, as follows:

X_8, X_{32}, X_{56}, X_{80} - Sector 4: Other Food and Kindred Products.
X_9, X_{33}, X_{57}, X_{81} - Sector 5: Other Non-Durables,
X_{10}, X_{34}, X_{58}, X_{82} - Sector 6: Farm Machinery,
X_{11}, X_{35}, X_{59}, X_{83} - Sector 7: Other Machinery,
X_{12}, X_{36}, X_{60}, X_{84} - Sector 8: Other Durables,
X_{13}, X_{37}, X_{61}, X_{85} - Sector 9: Regulated Industries,
X_{14}, X_{38}, X_{62}, X_{86} - Sector 10: Wholesale and Retail Trade,
X_{15}, X_{39}, X_{63}, X_{87} - Sector 11: Finance, Insurance, and Real Estate,
X_{16}, X_{40}, X_{64}, X_{88} - Sector 12: Other Services,
X_{17}, X_{41}, X_{65}, X_{89} - Sector 13: Construction and Mining.

Each of these activities is assumed to be located in the town shown in Figure 2. Each activity uses municipally treated water and discharges waste water to be treated by the municipal waste water treatment facility. Resource requirements per unit of each of these activities are shown in Table 2.

The water supply sector (X^S).

There are four activities in the water supply sector. Two of these activities, Water Treatment and Water Treatment II, represent treatment of stream water to meet commercial and residential requirements in the assumed municipality. Waste water treatment represents treatment of municipal waste water, while the fourth activity is Reservoir Storage, carried out in the reservoir shown in Figure 2. Resource requirements for each of these activities are shown in Table 3. The activities are defined as follows:

X_{12}, X_{42}, X_{66}, X_{90} - Water Treatment I, the activity which treats water withdrawn from stream I, water containing effluent from Pork Slaughter I. It is assumed that Pork Slaughter I discharges waste in quantities too small to affect the level of treatment costs at the municipal water treatment plant. The output of Water Treatment I is distributed among the various

Table 2. Resource requirements and net revenue per unit of output for any time period in non-agricultural production activities, application I[a]

	Activity					
	X_{61}, X_{30}, X_{54}, X_{78}, Pork Slaughter I (1000 hogs)	X_7, X_{31}, X_{55}, X_{79} Pork Slaughter II (1000 hogs)	X_8, X_{32}, X_{56}, X_{80} Other Food and Kindred, (exc. meat products) ($1000 VA)	X_9, X_{33}, X_{57}, X_{81} Other Non-Durables ($1000 VA)	X_{10}, X_{34}, X_{58}, X_{82} Farm Machinery ($1000 VA)	X_{11}, X_{35}, X_{59}, X_{83} Other Machinery ($1000 VA)
Labor (workers)	.1380	.1241	.1319	.1346	.1448	.1598
Land[b]	-	-	-	-	-	-
Water (gallons)						
stream I	8700	239,000				
stream II		-232,000				
treated water			119136.0	96970.3	59281.7	12135.2
waste water			-5170.0	-90109.3	-59281.7	-11216.4
Capital (dollars)			1572.2	1364.4	946.2	870.6
Net revenue (dollars)	13,130	13,130	1000	1000	1000	1000

[a] For sources and derivation of data, see text.

[b] The quality of land used is invariate with the volume of production. Therefore no coefficient need be included in the model.

Table 2. (Continued)

	Activity					
	X_{12}, X_{36}, X_{60}, X_{84} Other Durables ($1000 VA)	X_{13}, X_{37}, X_{61}, X_{85} Regulated Indistries ($1000 VA)	X_{14}, X_{38}, X_{62}, X_{86} Wholesale and Retail Trade ($1000 VA)	X_{15}, X_{39}, X_{63}, X_{87} Finances, Insurances and real estate ($1000 VA)	X_{16}, X_{40}, X_{64}, X_{88} Other Services ($1000 VA)	X_{17}, X_{41}, X_{65}, X_{89} Construction and Mining ($1000 VA)
Labor (workers)	.1343	.0819	.2100	.0510	.2021	.1412
Land	-	-	-	-	-	-
Water (gallons)						
stream I						
stream II						
treated water	80,268.2	522,978.4	8,276.8	2,608.2	31,667.3	1,261,768.9
waste water	-70,668.7	-108,312.7	-7,357.2	-2,412.6	-10,050.5	-883,582.8
Capital (dollars)	1,024.3	3,266.9	857.0	1,706.9	1,273.5	506.0
Net revenue (dollars)	1,000	1,000	1,000	1,000	1,000	1,000

commercial and residential water using activities in the town.

X_{19}, X_{43}, X_{67}, X_{91} - Water Treatment II, which treats water withdrawn from stream II, containing effluent water from Pork Slaughter II. The main difference between Water Treatment II and Water Treatment I is the level of treatment cost per unit of water. Water Treatment II shows a higher cost, reflecting the increased pollution load caused by the Pork Slaughter II activity.

The amount by which treatment costs are increased by the effluent of Pork Slaughter II is based upon \$175 per million gallons average cost for treatment including filtration, given in Seidel and Cleasby (79, p. 1522). It is assumed that Water Treatment II is ten per cent more costly than Water Treatment I, and that Water Treatment I costs equal the average cost shown above. Water Treatment II, therefore, is \$17.50 per million gallons more expensive than Water Treatment I.

In most water treatment facilities, some proportion of total output is "unaccounted-for" water, not distributed to customers (79, p. 1509). According to the Seidel and Cleasby survey, the most frequently reported proportion was in the range of ten per cent to fifteen per cent. For this study, the mid-point of this range, a twelve and one-half per cent loss before distribution, will be assumed. Therefore, for each 1,000 gallons treated, 1138.6 gallons must be withdrawn from the stream in both Water Treatment I and Water Treatment II.

X_{20}, X_{44}, X_{68}, X_{92} - Waste Water Treatment. This activity represents the treatment of sewage from residential and commercial activities in the town. The sewage effluent from the town is considered to be of the same quality regardless of the water's quality prior to initial treatment. Therefore, one activity is constructed to represent waste treatment under any configuration of activities and stream quality levels.

The capacity of each of the three water supply activities listed above represents the average capacity of that type of facility for all towns in Iowa of 2500 to 10,000 population in 1960. These capacities, expressed in gallons, were estimated based on data in the 1962 Inventory of Municipal Waste Facilities (97a) and the 1962 Inventory of Municipal Water Facilities (97b). A ratio estimate, treatment capacity per capita, was calculated and used to estimate treatment facility capacity in the hypothetical town of 10,000. The coefficient representing capacity used per gallon of water or waste water treated is unity, since capacity in each activity is expressed in gallons.

Labor and land are not included as variable resource requirements. Interview data for selected Iowa water treatment and sewage plants showed that over a broad range of output, a fixed amount of labor is required due to process automation. In the short run, therefore, labor requirements for each treatment operation are assumed to be invariate with output. Land requirements are considered also to be invariate for the water treatment and waste treatment activities.

X_{21} - Reservoir, a storage facility, which is also used for recreation. Reservoir capacity is discussed in a later section on the model's parameters. Stored water can be released during low-flow periods for flow augmentation, either to meet withdrawal requirements or to be used for pollution abatement. Recreation use and flow augmentation of reservoir water are competing uses to some degree, since a relatively stable water level is desirable for recreation, while flow augmentation implies a fluctuating water level.

Because storage is a transfer activity, it must appear in every time period. Therefore, X_{45}, X_{69}, and X_{93} are reservoir storage activities in periods 2, 3, and 4, respectively.

The residential sector (X^r)

The residential sector contains four activities representing some of the uses of water which do not normally produce market valued output. The four activities are defined as follows:

X_{22}, X_{46}, X_{70}, X_{94} - Residential Use, based on an average use of 79,000 gallons per residence per year (59, p. 1512). This activity does not represent water used by commercial activities in the assumed municipality.

Table 3. Resource requirements and net revenue per 1000 gallons of water or waste water treated

Resource	X_{18}, X_{42}, X_{66}, X_{90} Water Treatment I	X_{19}, X_{43}, X_{67}, X_{91} Water Treatment II	X_{20}, X_{44}, X_{68}, X_{92} Waste Water Treatment
Land (acres)[a]	0	0	0
Labor (workers)[a]	0	0	0
Water intake (gallons)	1,138.6[b] (Stream I)	1,138.6[b] (Stream II)	1,000.0
Capacity (gallons)	1,000.0	1,000.0	1,000
Net revenue (dollard)	0	-.175[c]	0

[a] The quantities of land and labor used are invariate with the volume of water treated. Therefore, no coefficient is required.

[b] Based on 12.5 per cent loss between intake and distribution (79, p. 1509).

[c] See accompanying discussion for source and derivation.

X_{23}, X_{47}, X_{71}, X_{95} - Recreation Use of Reservoir, which represents reservation of some proportion, in this model an assumed 90 per cent, of reservoir storage for recreational uses such as swimming, boating, or fishing.

X_{24}, X_{48}, X_{72}, X_{96} - Low Flow Protection, which is a feature of Iowa's permit system (48, sect. 455A.1). Reservation of a minimum amount of flow for nonregulated uses grants these uses maximum protection of right as long as there is water in the stream. Knowledge of the opportunity cost of reserving a quantity of water for low flow protection requires that it be explicitly recognized as an activity.

Constraint parameters

In the general model shown in Chapter V, the set of constraint parameters was defined to contain four vectors, b, $\overline{X}^P$, $\overline{X}^s$, and $\overline{X}^r$. The elements of the b vector represents available amounts of labor, water, and reservoir storage capacity in each time period, as well as the amount of land available annually to agricultural activities in the model. In Table 4, where the parameter values used in application I are shown, the elements of the b vector corresponding to labor, water, and reservoir storage parameters are designated b_{it}, $t = 1, \ldots, 4$, denoting that the parameter value varies among the four time periods.

Elements in $\overline{X}^P$ and $\overline{X}^s$, denoted by $\overline{X}_i$, $i = 6, \ldots, 22$, are shown only as annual amounts. These parameters specify the amount of available annual capacity in producing and water-supply activities. Elements of $\overline{X}^r$, which represent the minimum amounts of water reserved for residential, low flow, and recreation uses, are denoted by $\overline{X}_i$, $i = 23, 24, 25$.

The individual constraint parameters for application I were calculated as follows:

Water - all water in the model is initially in stream I (b_{1t}, $t = 1, \ldots, 4$). The available runoff was based on below normal, average, and above normal rainfalls (80, p. 6), assuming that the relationship between

Table 4. Resource parameters, application I.

Resource	Available amount Annual	Period 1	Period 2	Period 3	Period 4
b_1 stream 1 (gallons)					
above normal rainfall		1,453,525,336	1,161,429,336	462,485,334	406,848,000
normal rainfall		952,789,336	760,840,534	303,918,934	367,059,200
below normal rainfall		315,046,400	254,540,800	100,530.934	88,324,266
b_2 stream II (gallons)					
above normal rainfall		1,453,525,336	1,161,429,336	462,485,334	406,848,000
normal rainfall		952,789,336	760,840,534	303,918,934	267,059,200
below normal rainfall		315,046,400	254,540,800	100,530.934	88,324,266
b_3 labor (workers)	1,427				
b_4 reservoir (gallons)		252,896,964	252,896,964	252,896,964	252,896,964
b_5 land I (acres	500				
b_6 land II (acres)	500				
$\overline{X}_1$ feed lot capital ($)	32,200				
$\overline{X}_2$ corn capital ($)	44,330				
$\overline{X}_3$ soybean capital ($)	39,620				
$\overline{X}_4$ pork slaughter I capacity (carcasses)	478,400				

Table 4. (Continued)

Resource	Available amount Annual	Period I	Period 2	Period 3	Period 4
$\overline{X}_5$ pork slaughter II capacity (carcasses)	166,900				
$\overline{X}_6$ non-durable goods capital ($)	1,005,422				
$\overline{X}_7$ durable goods capital ($)	866,810				
$\overline{X}_8$ regulated industries capital ($)	3,743,972				
$\overline{X}_9$ wholesale and retail trade capital ($)	3,904,438				
$\overline{X}_{10}$ finance, insurance and real estate capital ($)	4,705,960				
$\overline{X}_{11}$ other services capital ($)	4,991,927				
$\overline{X}_{12}$ construction and mining capital ($)	928,240				
$\overline{X}_{13}$ water treatment capacity (gallons)	448,950.000				

Table 4. (Continued)

Resource	Available amount Annual	Period 1	Period 2	Period 3	Period 4
$\bar{X}_{14}$ waste water treatment capacity (gallons)	381,279,000				
$\bar{X}_{15}$ residential use (gallons)	266,862,000				
$\bar{X}_{16}$ protected low flow (gallons)		6,300,754	12,563,779	3,150,377	2,527,847
$\bar{X}_{17}$ recreation in reservoir (gallons)		12,650,000	12,650,000	12,650,000	12,650,000
$\bar{X}_{18}$ treated water (gallons)		0			
$\bar{X}_{19}$ waste water (gallons)		0			

annual rainfall and annual runoff can be expressed as

$$\log \text{(annual runoff)} = -3.1 + 2.6 \left[\log \text{(annual rainfall)}\right]$$ [8]

It is further assumed that runoff is distributed unevenly throughout the year, with 41.7 per cent appearing in period 2; 33.3 per cent in period 2; 13.3 per cent in period 3; and 11.7 per cent in period 4, based on information given by Bennion (6, p. 11). If a 20 square mile drainage area is assumed, the amount of available water with below normal, normal, and above normal rainfall is that shown in Table 2, Appendix A.

According to Shaw (80, p. 6), each of the three ranges of rainfall discussed above is equally probable. Of the three events, below normal rainfall is the event which would create situations most conducive to water scarcity. It is conditions arising in water scarcity which this model is designed to treat. Therefore, streamflow levels and crop water requirements in this application are those which result when rainfall on the drainage area of the stream is below normal.

Stream II (b_{2t}, t = 1,..., 4) is a transfer row showing movement of water from a higher to a lower quality supply as wastes from the Pork Slaughter II activity are discharged into Stream I. The value of b_{2t} in any period depends upon the amount of water initially available in that period in Stream I and upon the amount consumed by those activities which withdraw water from Stream I. These activities are as follows:

Cattle Feed Lot -	X_1, X_{25}, X_{49}, X_{73}
Corn I	X_2, X_{26}, X_{50}, X_{74}
Corn II	X_3, X_{27}, X_{51}, X_{75}
Soybeans I	X_4, X_{28}, X_{52}, X_{76}
Soybeans II	X_5, X_{29}, X_{53}, X_{77}
Pork Slaughter I	X_6, X_{30}, X_{54}, X_{78}

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[8] Merwin Dougal, Civil Engineering Department, Iowa State University, Ames, Iowa. Data on rainfall - runoff relationship. Private communication. July 3, 1969.

Pork Slaughter II	X_7, X_{31}, X_{55}, X_{79}
Water Treatment I	X_{18}, X_{42}, X_{66}, X_{90}
Reservoir	X_{21}, X_{45}, X_{69}, X_{93}
Low Flow Protection	X_{29}, X_{48}, X_{72}, X_{96}

Thus, for time period 1, b_{23} is given by the expression $b_{21} = b_{11} - \left[a_{11} X_1 + a_{12} X_2 + A_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + (a_{17} - a_{27}) X_7 + a_{1,21} X_{21} + a_{1,24} X_{24}\right]$. Applying this relation to the b_{21} row yields $a_{11} X_1 + a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + (a_{17} - 2a_{27}) X_7 + a_{1,18} X_{18} + a_{1,21} X_{21} + a_{1,24} X_{24} \quad b_{11}$, which becomes the row of the A matrix representing use of the Stream II resource. For each of the other time periods in the model, the form of the corresponding Stream II row is the same as that shown above.

Land (b_5, b_6) - for each soil type in the model, Tama silty clay loam and Clarion loam, 500 acres are hypothesized to be available and irrigable annually.[9] Land which is not irrigable due to unfavorable slope or erosion characteristics is not considered in the model.

Labor (b_3) - the annual labor resource, expressed in man-years, is 1,427 persons, calculated using the Iowa average employment in 1960 in urban places of 2,500 to 10,000 population, which is computed from data in the U. S. Census of population, 1960 (96, Table 70, p. 17-199). Labor is assumed to have no seasonal fluctuations, and is therefore expressed as an annual total.

Reservoir Capacity (b_{4t}, $t = 1, \ldots, 4$) - according to Schwab (77, p. 28), a survey of ten reservoirs in Iowa showed that those which contained sufficient storage to last through the 1934 drought had an average watershed area (acres): reservoir capacity (acre-feet) ratio of

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[9] 500 acres for crop production is not intended to reflect any actual configuration of land use. The quantity available here is hypothetical and may be varied at will by future users of this model.

3.3. This ratio is used to determine the storage requirements for a watershed area of 20 square miles, or 12,800 acres.

Each activity in X^P, the producing sector, and X^s, the water supply sector, is constrained by its available short-run production capacity. This annual capacity limit can be expressed either as a physical or monetary amount. The capacity constraint parameters, denoted as $\overline{X}_i$, used in application I were computed as follows:

Feedlot Capacity ($\overline{X}_1$) - this activity is assumed to have available enough capital to produce 1000 fed steers, each animal requiring $33.20 in capital, including land, buildings, and equipment (35, Table 35, p. 75).

Corn Production Capacity ($\overline{X}_2$) - it is assumed that this activity has sufficient capital to utilize all available land of both soil types in the model. On Tama loam, $44.52 per acre is required (51, Table 8.1, p. 213-214); on Clarion loam, $44.14 per acre is required (51, Table 8.1, pp. 213-214).

Soybean Production Capacity ($\overline{X}_3$) - soybean activities are also assumed to have sufficient capital to utilize all available land in the model. For soybean production, $39.23 per acre is required on Tama loam, and $38.99 per acre is required on Clarion loam (51, Table 8.2, pp. 215-216).

Pork Slaughter I and Pork Slaughter II ($\overline{X}_4$, $\overline{X}_5$) - each packing activity is limited to an annual capacity equal to production at the activity's assumed rate for 260 work days (52 weeks, 5 days per week). Pork Slaughter I operates at 230 carcasses per hour, or 1840 carcasses per work day. Its annual limit is therefore 476,400 carcasses. Pork Slaughter II operates at 310 carcasses per hour, or 2480 carcasses per work day. Its limit is therefore 644,800 carcasses per year.

Other Producing Activities ($\overline{X}_6$ through $\overline{X}_{11}$) - Capacities in activities X_8 through X_{17} are based on a capital-labor ratio computed as shown in Table 19, Appendix A, from Barnard (3). The capital stock in each major industry group is shown in Table 21, Appendix A. In computing available capital stock, some groups of activities are aggregated because 1960 employment data in sectors corresponding to those defined in this model do not exist for urban places of 2,500 to 10,000 population (96, Table 70, p. 16-199). The capital stock in a sector activity or a major industry group of sector activities is calculated by the equation

$$\text{Capital Stock}_i = \left(\sum_{j=1}^{n} e_j \frac{\text{Capital}_j}{\text{Worker}_j}\right) \text{Workers}_i \text{ (1960)}.$$

The terms denoted by subscript $_j$ refer to sectors as defined in this study; the terms denoted by subscript i refer to major industry groups for which employment data are published specific to urban places of 2,500 to 10,000 population. The sum enclosed in parentheses represents a weighted average of capital per worker ratios in those sectors which must be aggregated to correspond with published employment data. The weights, e_j, are of the form $e_j = \frac{\text{Employment}_j}{\sum_{j=1}^{n} \text{Employment}_j}$, where n is the number of sectors included in the major industry groups. Worker_i (1960), the employment in the model in the ith major industry group, was calculated by allocating total labor force to the several industry groups in proportion to that industry group's share of 1960 total employment in urban places, 2,500 to 10,000 population.

Water Treatment Capacity ($\overline{X}_{12}$) - the sum of the outputs of both treatment activities is constrained to be no greater than the capacity of the plant. The assumed capacity is based on the average production in treatment plants serving populations of 5,000-10,000, as estimated by Seidel and Cleasby (52, Table 2, p. 1509). The average, 123 gallons per capita per day, is equivalent to 1.23 million gallons per day for a city of 10,000, or 448.95 million gallons per year.

Waste Water Treatment Capacity ($\overline{X}_{14}$) - a ratio of gallons of treatment capacity per capita was calculated for 48 places in Iowa of 2,500 to 10,000 population, based on data given in the 1962 Inventory of Waste Facilities (97b). Estimated waste treatment capacity is 104.46 gallons per capita per day. Applying this average to a town of 10,000 population yields an estimate of 381,279,000 gallons annual capacity.

Residential Use ($\overline{X}_{15}$) - This constraint represents the amount of water which is reserved in the model for human consumption. In order to avoid reserving water for use by commercial or industrial users in the hypothetical town, average residential use, rather than per capita total water use, is the basis of estimated requirements. The average given by Seidel and Cleasby (79, Table 5, p. 1512), for treatment facilities with a daily output of 1.0 to 2.0 million gallons, is 79,000 gallons per year per residence.

The number of residences in the hypothetical town was estimated by using the 1960 average population per household, 2.96, in urban places 2,500 to 10,000 population (96, Table 71, p. 200). It is assumed that there is only one household per residence, so that there are an estimated 3.378 residences in the town, requiring 266,862,000 gallons per year.

Protected Low Flow ($\overline{X}_{16}$) - according to Hines (43, p. 44), the protected low flow in Iowa streams is generally set at that level of flow expected to be exceeded 84 per cent of the time between April and September. In applying the above standard to individual streams, protected flow may be increased or decreased according to public interest (43, p. 44). However, since the 84 per cent standard is the basis of the protected flow standard, the amount reserved for protected flow in this model is similarly calculated, in the following manner.

Work by Beer has shown that the distribution of annual rainfall approximates a log normal distribution.[10] The parameters of this distribution, the mean and variance, are estimated from annual rainfall data for a period of 96 years (104). The sample mean is $\overline{X} = 1.493$, and the sample variance is $s^2 = 0.0043$. Given the relationship

log (annual runoff) = -3.1 + 2.6 log (annual rainfall), the distribution of log (annual runoff) can be specified as normal, with an estimated mean of

$$\overline{y} = -3.1 + 2.6\ (\overline{X}) = 0.7418$$

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[10] Craig Beer, Agricultural Engineering Department, Iowa State University, Ames, Iowa. Data on the statistical distribution of rainfall. Private communication. July 10, 1969.

and an estimated variance of $s^2_y = 2.6^2 (s^2_x) = 2.9068$.
Based on the estimates above, the annual runoff which can be expected to be exceeded 84 per cent of the time (y^*) is approximated by (assuming the expected value of the error of the estimate to be zero)

$$y^* = \bar{y} - z_{.84} \quad s_y \text{ (71, p. 90)},$$

where $-z_{.84}$ is the point on the normal distribution such that $P\left[z \geqslant -z_{.84} \right] = .84$ (71, p. 517). The tabulated z-value is $z = -1.00$ (71, Appendix 5, p. 517). Using this z-value, y^* is given by

$y^* = 0.7418 \quad -1.00 (1.7049) = -0.9631$ or .09185 inches of annual runoff.

Over the 20 square mile drainage basin assumed this runoff is equal to an annual flow of 37,729,066 gallons, of which 16.7 per cent (6, p. 11), or 6,300,754 gallons, occurs in period I, during the month of April. During period 2, 33.3 per cent (6, p. 11), or 12,563,779 gallons, occurs. During period 3, 13.3 per cent (6, p. 11), or 3,150,377 gallons, occurs, and during the month of September in period 4, 6.7 per cent (6, p. 11) appears, or 2,527.847 gallons.

Recreation in Reservoir ($\bar{X}_{17}$) - the amount of water reserved in the reservoir for recreation use is assumed to be 90 per cent of reservoir capacity.

Four additional constraints are imposed on the model. The amounts of treated water and waste water in the model are constrained to be zero, indicating no storage of treated water or sewage. In addition, Pork Slaughter II is constrained to be at a level greater than or equal to 478,400 carcasses annually, since Pork Slaughter I and Pork Slaughter II are assumed to be mutually exclusive up to the maximum available from Pork Slaughter I. The fourth constraint insures that the sum of output in Pork Slaughter I and Pork Slaughter II must be no greater than 648,400 carcasses, the maximum amount of production possible.

Application II: an Existing Water Use Situation

Application II represents the analysis of water use from a shallow sand and gravel aquifer by industrial, commercial, and residential uses in a town whose 1960 population was 4,350. The water supply and the activities which withdraw from it are not atypical of Iowa. Application II illustrates the problems encountered in applying the model developed in this study to real situations. In estimating technical coefficients and parameters for use in the model, accuracy increases as the sample size on which the estimate is based increases. Inasmuch as the resources available in governmental agencies for the collection of large amounts of primary data may be limited, an effort was made in this analysis to utilize secondary sources instead of primary sampling in estimating as many coefficients and parameters as possible.

The activities are located along a river with estimated average flow of 210 cubic feet per second. No activity considered in the model withdraws water from this stream; the only use of the stream at this point is for effluent carriage. There are no significant uses of the stream for at least 10 miles downstream[11] so that this river is not considered as a water supply in the model.

In this application, there are no activities whose water requirements fluctuate seasonally. Also, the water supply under study would not be expected to show significant seasonal variation in quantity.[12] Therefore, only a single, annual time period is considered.

Activities

A number of activities in application II are identical to activities in application I. Where this is the case, reference is made to that activity's definition in the discussion of application I above; where the activity is unique to application II, it is defined in paragraphs below.

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[11] Richard G. Bullard, State Water Commissioner, Des Moines, Iowa. Data on water use in Iowa. Private communication. June 30, 1969.

[12] Dr. Lyle V. Sendlein, Department of Earth Sciences, Iowa State University, Ames, Iowa. Data from a study in progress of surficial aquifer in a southwestern Iowa river bottom. Private communication. July 7, 1969.

Resource and objective function coefficients for these activities appear in Table 5.

The producing sector (X^P) There are no crop agricultural activities in application II. A soil survey of the area under consideration (97) shows very little land of a slope and soil type which would permit irrigation from the aquifer water supply under study. Therefore, only nonagricultural producing activities are considered. These activities are defined as follows:

X_1 - Pork Slaughter, which has coefficients of resource use identical with Pork Slaughter II in application I. In pumping its water from an aquifer and discharging it into a stream, this activity moves the water which it does not consume into a different water supply. For this reason, only the gross water intake coefficient is shown.

Activities X_2 through X_{10} are represented by sectors, which are defined as shown in Table 4, Appendix A.

X_2 - sector 4: Other Food and Kindred Products,
X_3 - sector 5: Other Non-durables,
X_4 - sector 6: Farm Machinery,
X_5 - sector 7: Other Machinery,
X_6 - sector 8: Other Durables,
X_7 - sector 9: Regulated Industries,
X_8 - sector 10: Wholesale and Retail Trade,
X_9 - sector 11: Finance, Insurance, and Real Estate,
X_{10} - sector 12: Other Services,
X_{11} - sector 13: Construction and Mining.

With the exception of X_1, all these activities are located in the town, and are dependent upon municipal facilities for water supply and waste water treatment.

The water-supply sector (X^s) There are only two activities in the water supply sector, a water treatment activity and a waste water treatment activity. Data on which the resource coefficients of these two activities were calculated were gathered in personal interview with the

Table 5. Resource requirements and net revenue per unit of output for activities, application II

	Activities				
	X_1 Pork Slaughter (1,000 hogs)	X_2 Other Food and Kindred ($1,000 VA)	X_3 Other Non-durables ($1,000 VA)	X_4 Farm Machinery ($1,000 VA)	X_5 Other Machinery ($1,000 VA)
Labor (workers)	.1241	.1319	.1346	.1448	.1598
Water (gallons)					
from aquifer	239,000				
treated water		119,136.0	96,970.3	59,281.7	12,135.2
waste water		-5,170.0	-90,109.0	-59,281.7	-11,216.4
Capital (dollars)		1,572.2	1,364.4	946.2	870.6
Net revenue (dollars)	13,130	1,000	1,000	1,000	1,000

Table 5. (Continued)

	Activities					
	X_6 Other Durables ($1,000 VA)	X_7 Regulated Industries ($1,000 VA)	X_8 Wholesale and Retail Trade ($1,000 VA)	X_9 Finances, Insurances, & Real Estate ($1,000 VA)	X_{10} Other Services ($1,000 VA)	X_{11} Construction and Mining ($1,000 VA)
Labor (workers)	.1343	.0819	.2100	.0510	.2021	.1412
Water (gallons) from aquifer						
treated water	80,268.2	522,978.4	8,276.8	2,608.2	31,667.3	1,261,768.9
waste water	-70,668.7	-108,312.7	-7,357.2	-2,412.6	-10,050.5	-883,582.8
Capital (dollars)	1,024.3	3,266.9	857.0	1,706.9	1,273.5	506.0
Net revenue (dollars)	1,000	1,000	1,000	1,000	1,000	1,000

plant supervisory personnel.

X_{12} - Water Treatment, withdraws water from the same aquifer system on which the Pork Slaughter activity depends. This water is then distributed to activities within the town.

X_{13} - Waste Water Treatment, which treats waste effluent from municipal users, discharging treated water into the stream on which the town is located. Since the stream is not considered as a source in this study, the discharge from the waste treatment activity is not shown.

The residential sector (X^r) The residential sector contains only a residential use activity. There is no protected low flow activity in this model. The Iowa Water Commissioner had not found any low flow protection necessary, since there are no withdrawals being made from the stream in the reach under study.[13] The residential use activity is defined as follows:

X_{14} - Residential Use, which requires an estimated 79,000 gallons per residence per year (79; Table 5, p. 1512).

Constraint Parameters

The general set of constraint parameters consists of four vectors, b, $\overline{X}^p$, $\overline{X}^s$, and $\overline{X}^r$. In application II, the b vector contains two elements, which express the annual amount of water available from the aquifer and the annual labor supply. $\overline{X}^p$ and $\overline{X}^s$ express the annual capacity of each of the producing activities and water supply activities, respectively. $\overline{X}^r$ expresses the amount of water reserved in the aquifer for residential use. The individual constraint parameters, listed in Table 6, are defined as follows:

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[13] Richard G. Bullard, State Water Commissioner, Des Moines, Iowa. Data on water use in Iowa. Private communication. July 3, 1969.

Water (b_1) - the maximum safe yield[14] in the municipal well field was estimated at the time the wells were installed to be two million gallons per day, or an annual amount of 730 million gallons per year. This estimated capacity, however, does not include the water available from the aquifer at points other than the municipal well field.

An estimate of the total flow in the aquifer was made[15] based on the following relation:

$$Q = K \cdot I \cdot A \quad (56, \text{ p. } 81),$$

where Q is total flow in the aquifer in gallons per day; K is a constant describing the permeability of the aquifer, or its ability to transmit water, in gallons per square foot per day; I is the gradient in the aquifer, in feet per horizontal foot. A represents the cross-sectional area of the water-bearing material, in square feet. Permeability (K) was assumed to be 4000 gallons per day per square foot, based on tests made in similar aquifer systems. Gradient (I) is estimated to be 13.5 feet per 1000 feet. The aquifer under study lies in a river valley, and the largest component of flow in the aquifer is from the valley wall to the stream bed. This gradient constant represents the gradient of flow in that direction, perpendicular to the direction of stream flow. A, the cross-sectional area of the water-bearing material, is estimated to be 158,000 square feet, since the bed of sand and gravel is approximately one mile wide and thirty feet deep. The quantity of water flowing in the system described above, according to the formula $Q = K \cdot I \cdot A$, is approximately 8.5 million gallons per day.

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[14] Maximum safe yield is that rate at which water can be withdrawn from an aquifer without exceeding the rate at which the aquifer is recharged. To exceed the recharge rate in withdrawal is to incur an overdraft, which may damage the medium of the aquifer, permanently impairing the storage or transmission characteristic of the aquifer (56, pp. 101-102).

[15] Dr. Lyle V. A. Sendlein, Department of Earth Sciences, Iowa State University, Ames, Iowa. Data from a study in progress of surficial aquifers in a southwestern Iowa river basin. Private communication. June 30, 1969.

Table 6. Resource parameters, application II[a]

	Resource	Parameter Value
b_1	aquifer (gallons)	2,372,500,000
b_2	labor (workers)	2,014
$\overline{X}_1$	pork slaughter capacity (carcasses)	644,800
$\overline{X}_2$	manufacturing capital (dollars)	2,669,952
$\overline{X}_3$	regulated industries capital (dollars)	3,896,166
$\overline{X}_4$	wholesale and retail trade capital (dollars)	2,348,734
$\overline{X}_5$	finance, real estate and insurance capital (dollars)	3,991,953
$\overline{X}_6$	other services capital (dollars)	2,182,688
$\overline{X}_7$	construction and mining capital (dollars)	616,640
$\overline{X}_8$	water treatment capacity (gallons)	365,000,000
$\overline{X}_9$	waste water treatment capacity (gallons)	182,500,000
$\overline{X}_{10}$	residential use (gallons)	133,431,000

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[a] Refer to text for sources and derivation.

A second estimate of capacity was made, based on base flow[16] in the stream flowing through the valley in which the aquifer is located. After an extended period of little or no rainfall, the flow in the river, which comes largely from ground water sources, represents a portion of the water flowing in the aquifer. Analysis of stream flow records in the basin and particular hydrologic conditions near the point of study in the stream yielded a preliminary estimate of base flow of 10 cubic feet per second, or approximately 6.5 million gallons per day. Thus, all withdrawals from the aquifer may total as much as 6.5 million gallons per day without causing flow in the stream to disappear.

Inasmuch as the disappearance of stream flow may have serious effects on downstream uses, the limiting capacity of the aquifer will be assumed to be 6.5 million gallons per day, or 2372.5 million gallons per year.

Labor (b_2) - Defining the labor force available to the activities in this model is a difficult task, since there are few indicators of how many people located outside the municipal boundary travel to town to work. In this study, it is assumed that the available labor force is 2014 workers. Labor force was estimated by calculating the proportion of 1960 county employment which was located in the town, for seven major industry groups as shown in Table 22, Appendix B. These proportions were then applied to estimates of 1967 county employment, by the same seven industry groups. These employment estimates were made by Dr. Marvin Julius, of the Department of Economics, Iowa State University. The resulting employment estimates are shown in Table 26, Appendix B. The total of employment in these industry groups is the assumed labor force.

Pork Slaughter Capacity ($\overline{X}_1$) - this parameter is the number of carcasses which could be processed annually at a rate of 310 carcasses per hour, which is 644,800 carcasses.

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[16] Base flow is that flow in a stream which originates not as surface runoff, but as inflow from an aquifer (56, p. 39).

Capacity in other producing activities ($\overline{X}_2$ through $\overline{X}_{10}$) - the amount of available capital stock for each of these activities is calculated by the same method used to calculate this parameter for these same activities in application I above. The relationship used is

$$\text{Capital Stock}_i = \left(\sum_{j=1}^{n} e_j \frac{\text{capital}_j}{\text{worker}_j}\right) \text{workers}_i \text{ (1967), where}$$

the term enclosed in parentheses represents a weighted average of capital per worker ratios in those sectors defined in this study (Table 19, Appendix A) which must be aggregated in order to compare with employment data for the major industry groups shown in Table 1, Appendix B. The term, workers_i (1967), is the estimated employment in the _i_th major industry group, as shown in Table 22, Appendix B. The calculated capital stocks are shown in Table 23, Appendix B.

Water Treatment Capacity ($\overline{X}_{11}$) - the capacity of the municipal treatment plant is one million gallons per day, or 365 million gallons per year.

Waste Water Treatment ($\overline{X}_{12}$) - the capacity of the waste treatment facility is 500,000 gallons per day, or 182.5 million gallons per year.

Residential Use ($\overline{X}_{13}$) - the amount of water reserved for residential use is based on an estimated population of 5000 and an average annual requirement of 79,000 gallons per residence (79, Table 5, p. 1512). Assuming 2.96 persons per household (96, Table 71, p. 200) and one household per residence, there are approximately 1689 residences requiring 79,000 gallons each per year, or 133,431,000 gallons per year.

Based on the data described in this chapter, the two model applications were solved. The results of these solutions, as well as a summary of the study, are contained in the following chapter.

CHAPTER SEVEN: RESULTS OF MODEL SOLUTIONS

Solution of the model, in either application I or application II, yields a vector of optimum activity levels which is unique to the particular set of constraints and parameters in the problem. In general, a change in either the constraints or parameters of the problem will cause the solution vector to be changed. Therefore, by solving each application repeatedly under various sets of constraints or parameters, certain comparisons can be made which will be valuable in reaching a conclusion with respect to the hypothesis developed in Chapter IV.

The initial solution of each application was reached using the data and relationships described in Chapter VI. This solution is the basis for subsequent comparisons within the framework of each application. The initial solution determines optimum activity levels, optimum water use and allocations, and the optimum value of marginal product of water. The value of the objective function in this solution represents the value added in production when a scarce water resource is optimally allocated.

The second solution of each application approximates the actual pattern of water use by forcing the level of each producing activity to be equal to the estimated actual output of that activity in the year which the particular data used represent. This solution determines a new value of the objective function for each application which is less than or equal to the value determined in the initial solution. These two values define a range over which the permit system could, if properly operated, improve the value added in production which utilizes the particular water sources under study. This range of values of the objective function indicates the potential gain to the hydrologic area from optimum allocation under the permit system. Each of the solutions listed above is described in this chapter; a final, summary section, which incorporates suggestions for further study, concludes the analysis.

Results of Application I

Tables 7a, 7b, and 7c show the results of the initial solution of application I. This solution is based on the data described in Chapter VI. Table 7a shows water to be a constraining resource in this situation, but not throughout the year. The supply of water in Stream I is exhausted only in Periods 3 and 4, which are low rainfall periods.[1] This shortage serves to provide in this initial optimum solution, a baseline against which the comparisons previously discussed can be made.

Before any comparisons are undertaken, several points of interest should be noted in the initial solution. First, the scarce water resource is Stream I, which carries the relatively unpolluted effluent of the Pork Slaughter I activity. The relative abundance of Stream II, which carries the more polluted effluent of Pork Slaughter II, indicates that the scarcity arises from the degradation of water in Stream I. The shadow price of this water, \$0.15 per thousand gallons, represents the value of marginal product of Stream I water. This value can be interpreted in several ways within the restrictions of the model. The \$0.15 is the dollar benefit which would be realized from every additional thousand gallons of Pork Slaughter II effluent returned to the original quality of the stream, whether by the pork processor or by the town. \$0.15 is also the opportunity cost associated with the loss of one thousand gallons of less polluted water. This cost, as well as the municipal treatment cost and the cost of adequate treatment at the source of pollution, are data which can be used in analysis of this production diseconomy. Such an analysis is beyond the scope of this thesis and is not attempted here.[2]

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[1] See Figure 3, Chapter VI for a depiction of rainfall by periods.

[2] For discussion of an analytical technique which would apply to this particular external effect, see Turvey (94a).

Table 7a. Initial solution of application I, seasonal resource use

Objective function value: $17,480,639.99

Resource	Resource used (gallons)	Unused Resource (gallons)	Shadow Price (dollars)
Period 1:			
Stream I	267,339,726	47,706,674	0
Stream II	229,913,857	85,132,543	0
Reservoir Capacity	12,650,000	0	0.00015
Period 2:			
Stream I	90,240,024	164,300,775	0
Stream II	77,764,735	177,676,065	0
Reservoir Capacity	12,650,000	0	0.00015
Period 3:			
Stream I	100,404,934	0	0.00015
Stream II	95,823,946	4,670,988	0
Reservoir Capacity	12,650,000	0	0.00015
Period 4:			
Stream I	88,324,266	0	0.00015
Stream II	76,656,645	11,667,620	0
Reservior Capacity	12,650,000	0	0.00015

Table 7b. Initial solution of application I, nonseasonal resource use

Resource	Resource used	Unused Resource	Shadow Price (dollars)
Labor (workers)	1,427	0	4,687.46
Land I (acres)	0	500	0
Land II (acres)	0	500	0
Cattle Feed Lot Capital (dollars)	0	33,200	0
Corn Capital (dollars)	0	44,330	0
Soybean Capital (dollars)	0	39,260	0
Pork Slaughter I Capacity (carcasses)	476,800	0	12.48
Pork Slaughter II Capacity (carcasses)	168,000	0	12.55
Non-durable Goods Capital (dollars)	1,005,422	0	0.16373
Durable Goods Capital (dollars)	866,810	0	0.26142
Regulated Industries Capital (dollars)	0	3,743,972	0
Wholesale and Retail Trade Capital (dollars)	669,641	3,234,797	0
Finance, Real Estate and Insurance Capital (dollars)	4,705,960	0	0.44
Other Services Capital (dollars)	4,491,927	0	0.01
Construction and Mining Capital (dollars)	0	928,240	0
Water Treatment Capacity (gallons)	448,950,000	0	0.001
Waste Water Treatment Capacity (gallons)	381,279,000	0	0.001
Additional Water Treatment Capacity required (gallons)	99,550,400		
Additional Waste Water Treatment Capacity required (gallons)	23,147,800		

Table 7c. Initial solution of application I, optimum activity levels

Seasonal Activities:	Activity Level	Reduced Revenue
Period 1:		
Cattle Feed Lot	0	5,823.07
Pork Slaughter I (carcasses)	237,752	0
Pork Slaughter II (carcasses)	80,659	0
Water Treatment I (gallons)	215,496,000	0
Water Treatment II (gallons)	2,128,075	0.175
Waste Water Treatment (gallons)	107,736,000	0
Storage (gallons)	0	0.00015
Recreation (gallons)	12,625,000	0.00015
Residential Use (gallons)	89,959,632	0.00015
Low Flow (gallons)	629,989,000	0
Period 2:		
Corn I (bushels)	0	879.49
Corn II (bushels)	0	1,020.16
Soybeans I (bushels)	0	2,940.72
Soybeans II (bushels)	0	3,502.22
Pork Slaughter I (carcasses)	76,584	0
Pork Slaughter II (carcasses)	26,886	0
Water Treatment I (gallons)	71,122,650	0
Water Treatment II (gallons)	709,350	0.175
Waste Water Treatment (gallons)	35,912,000	0
Storage (gallons)	100,000	0
Recreation (gallons)	12,625,000	0.00015
Residential Use (gallons)	30,313,089	0.00015
Low Flow (gallons)	1,259,979	0
Period 3:		
Corn I (bushels)	0	223.25
Corn II (bushels)	0	223.25
Soybeans I (bushels)	0	1,018.87
Soybeans II (bushels)	0	1,206.38
Pork Slaughter I (carcasses)	85,840	0
Pork Slaughter II (carcasses)	33,608	0
Water Treatment I (gallons)	80,196,530	0
Water Treatment II (gallons)	9,593,470	0
Waste Water Treatment (gallons)	86,657,000	0
Storage (gallons)	0	0.00015
Recreation (gallons)	12,525,000	0.00015
Residential Use (gallons)	89,790,000	0.00015
Low Flow (gallons)	504,044	0

Table 7c. (Continued)

Seasonal Activities:	Activity Level	Reduced Revenue
Period 4:		
Corn I (bushels)	0	364.00
Corn II (bushels)	0	504.66
Soybeans I (bushels)	0	2,191.38
Soybeans II (bushels)	0	2,566.48
Pork Slaughter I (carcasses)	76,584	0
Pork Slaughter II (carcasses)	26,886	0
Water Treatment I (gallons)	71,122,650	0
Water Treatment II (gallons)	709,350	0
Waste Water Treatment (gallons)	42,145,810	0
Recreation (gallons)	12,625,000	0.00015
Residential Use (gallons)	56,829,164	0.00015
Low Flow (gallons)	251,890	0
Non-seasonal Activities:		
Other Food & Kindred	$ 639,500	0
Other Non-durable Goods	0	$ 41.41
Farm Machinery	0	$ 44.66
Other Machinery	$ 995,646	0
Other Durable Goods	0	$ 48.24
Regulated Industries	0	$ 15.19
Wholesale and Retail Trade	$ 781,400	0
Finance, Insurance, and Real Estate	$2,757,000	4.82
Other Services	$3,919,848	0
Construction and Mining	0	$1,807.22

The second point of interest is that the initial solution shows that two of the reserved water uses, residential use and recreation, show a "reduced revenue"[3] value of $0.15 per thousand gallons reserved. This is consistent with the fact that water reserved for these uses is withdrawn from Stream I, and is equivalent, therefore, to a reduction in Stream I flow. The low flow protection activity, however, does not "cost" anything in terms of the objective function value, since low flow can be reserved from Stream II, which is abundant. In situations of water scarcity, the reduction of reserved use levels would achieve the same increase in the objective function value as an increase in water supply, and should be considered explicitly as an alternative to the development of a new supply.

It is also of interest to note the presence to the Storage activity in period 2, representing the transfer of Stream I water from period 2 into period 3 for subsequent use. Optimal allocation of a scarce water resource may require, in just the fashion represented in the model, that water be stored in times of adequate flow to be used in later time periods. The need for such storage would arise from differences in the time pattern of water requirements and the pattern of seasonal water availability. It can also be seen that the discharge of this stored water in period 3 has decreased the amount of water available for recreation in period 3, during which the stored water is used. This drawdown of the reservoir level illustrates the conflict between recreation and intertemporal water transfer discussed in Chapter VI, and points to the possibility of some point of intersection between the demand for water in its recreational role and the demand for water in its role as an input to production. If a positive value in the objective function could be established for recreation, comparable to the

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[3] "Reduced revenue", shown in Table 7c, denotes the amount by which the value of the objective function would be decreased if a non-basis activity were included in the basis at unit level. Conversely, it shows the increase in the objective function due to reduction of any activity which is forced into the solution at a minimum level.

positive values of each producing activity, the model would indicate this optimum point of intersection, at which water would be of equal value in each role.

A modification was required in the model in both application I and application II. It was necessary to introduce real disposal activities[4] into each matrix which correspond to the constraints on water treatment capacity and waste water treatment capacity. In both applications, these capacities are not sufficient to process the entire water supply, which must be done if water is to be exhausted. The effect of a disposal activity associated with either treatment capacity constraint is to indicate, by the optimum level of the disposal activity, how much additional capacity is required to treat all available water, either initially or as waste water. The additional capacity required for treatment in the initial solution of application I is shown in Table 7b.

The second solution of application I is made subject to a set of bounds, one for each producing activity in the model. These bounds approximate the outputs which might have resulted in each of the producing activities in 1960, the year the data represent, had this hypothetical situation existed. Each agricultural activity and both Pork Slaughter processes are bound at their maximum levels determined by the smallest value which land or capital would allow in each case. The remainder of the producing activities, the aggregate sectors, were bound at the level which would result if the remaining labor in the model were distributed as it was distributed among the same sectors in Iowa in 1960. (66, Table 33, p. 131 shows this distribution). Table 8 shows the distribution of labor and the pertinent output bound fixed for each producing activity. Seasonal activities are not restricted seasonally, but only in total so that the model is free to allocate production among time periods.

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[4] A disposal activity is represented by a vector X_j in which all elements except one are zero. The single non-zero element, which has a value of minus one, is in the row representing the resource to which the disposal activity corresponds.

Table 8. Distribution of labor force among producing activities; output bounds used in application I

Activity	Labor used (workers)	Output with maximum labor use[a]	Maximum output is[b]
Cattle Feed Lot	5	1000 hd.[c]	1000 hd.
Corn I	0.6	24,310 bu.[c]	24,310 bu.
Corn II	0.5	22,730 bu.[c]	22,730 bu.
Soybeans I	0.7	8,550 bu.[c]	8,550 bu.
Soybeans II	0.5	7,530 bu.[c]	7,530 bu.
Pork Slaughter I	65.8	476,000 carcasses[c]	476,800 carcasses
Pork Slaughter II	20.8	168,000 carcasses[c]	168,000 carcasses
Other Food and Kindred products	57.2	$ 433,666	$ 350,126[c]
Other Non-durable Goods	70.6	$ 524,517	$ 333,446[c]
Farm Machinery	42.0	$ 290,055	$ 144,926[c]
Other Machinery	65.6	$ 410,513	$ 242,639[c]
Other Durable Goods	84.9	$ 632,167	$ 505,971[c]
Regulated Industries	127.0	$1,550,672	$1,146,032[c]
Wholesale and Retail Trade	391.5	$1,864,286[c]	$4,555,937
Finance, Insurance, and Real Estate	73.2	$1,435,294[c]	$2,757,021
Other Services	311.1	$1,539,337[c]	$3,919,848
Construction and Mining	109.3	$ 774,079[c]	$1,834,466

[a] This is the output which would be achieved if all allocated labor were used.

[b] This is the maximum output which either capacity or capital will allow.

[c] These values were chosen as output bounds.

In setting output bounds, it was necessary to compare the production which would result in any activity from total use of allocated labor with the maximum production possible given that activity's capital stock. In order to avoid infeasibility[5], the lower of these two maxima was chosen as the output bound in the second solution, as indicated in Table 8.

The activity level and levels of resource use in the second solution are shown in Table 9a, 9b, and 9c. Comparison of the first two solutions shows immediately that the initial solution makes far more efficient use of the resources in the model. The value of the objective function of the initial solution is \$3,073,291.06 higher than the value of the objective function in the second solution. This difference in the value of output in the two situations under consideration shows, within the limitations of the model, how much greater benefit would be received in the model area if scarce resources were reallocated optimally among the alternative producers of the model.

Table 9a also shows that water from Stream I is the constraining resource in this solution and that this scarcity occurs only in periods 3 and 4. This table further shows the shadow price of water to be \$0.15 per thousand gallons, which is equal to the shadow price of water in the previous solution. Thus, in this situation and over the range of these two solutions, the shadow price of water does not vary, even though the allocation of the resource and the value of output resulting from its use may vary widely.

These two solutions have shown, with certain restrictions to be discussed in a later section, the magnitude of the increase in total value of production which might be realized if scarce resources were optimally allocated. The upper end of a range is therefore constructed over which the permit system might increase the value of production which utilizes this stream as a source of water input.

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[5] An infeasible solution is one in which one or more of the constraints in the linear program cannot be met.

Table 9a. Second solution of application I; seasonal resource use

Objective function value: $14,407,348.93

Resource	Resource used (gallons)	Unused Resource (gallons)	Shadow Price (dollars)
Period 1:			
Stream I	269,149,726	45,896,674	0
Stream II	231,723,857	83,322,543	0
Reservoir Capacity	12,650,000	0	.00015
Period 2:			
Stream I	90,268,624	164,272,176	0
Stream II	77,793,334	176,747,465	0
Reservoir Capacity	12,650,000	0	.00015
Period 3:			
Stream I	100,494,934	0	.00015
Stream II	95,823,946	4,670,988	0
Reservoir Capacity	12,650,000	0	.00015
Period 4:			
Stream I	88,324,266	0	.00015
Stream II	76,656,646	11,667,620	0
Reservoir Capacity	12,650,000	0	.00015

Table 9b. Second solution of application I, nonseasonal resource use

Resource	Resource Used	Unused Resource	Shadow Price (dollars)
Labor (workers)	1,290	137	0
Land I (acres	500	0	___[a]
Land II (acres)	500	0	___[a]
Cattle Feed Lot Capital (dollars)	33,200	0	___[a]
Corn Capital (dollars)	22,075	22,255	0
Soybean Capital (dollars)	19,471	20,149	0
Pork Slaughter I Capacity (carcasses)	476,800	0	___[a]
Pork Slaughter II Capacity (carcasses)	168,000	0	___[a]
Non-durable Goods Capital (dollars)	1,005,422	0	___[a]
Durable Goods Capital (dollars)	866,637	173	0
Regulated Industries Capital (dollars)	3,743,972	0	___[a]
Wholesale and Retail Trade Capital (dollars)	1,597,693	2,306,745	0
Finance, Insurance, and Real Estate Capital (dollars)	2,449,903	2,256,057	0
Other Services Capital (dollars)	1,960,346	3,031,581	0
Construction and Mining Capital (dollars)	391,684	536,556	0
Water Treatment Capacity (gallons)	448,950,000	0	.00083
Waste Water Treatment Capacity (gallons)	381,279,000	0	.001
Additional Water Treatment Capacity required (gallons)	116,042,666		
Additional Waste Water Treatment Capacity required (gallons)	161,298,000		

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[a] No shadow price is given in the solution for these resources because they were not entirely consumed; in each case the unconsumed portion was less than 0.001 units. This difference disappears in rounding, but is sufficient to cause a zero shadow price.

Table 9c. Second solution of application I, optimum activity levels[a]

Activities	Activity Level
Cattle Feed Lot (head)	1,000
Corn I (bushels)	24,310
Corn II (bushels)	22,730
Soybeans I (bushels)	8,550
Soybeans II (bushels)	7,350
Pork Slaughter I (carcasses)	476,800
Pork Slaughter II (carcasses)	168,000
Water Treatment I (gallons)	438,637,177
Water Treatment II (gallons)	10,302,822
Waste Water Treatment (gallons)	381,279,000
Storage (gallons)	100,000
Recreation (gallons)	50,500,000
Residential Use (gallons)	266,891,866
Low Flow (gallons)	2,645,902
Other Food & Kindred Products (dollars)	350,126
Other Non-durable Goods (dollars)	333,446
Farm Machinery (dollars)	144,926
Other Machinery (dollars)	242,639
Other Durable Goods (dollars)	505,971
Regulated Industries (dollars)	1,146,032
Wholesale and Retail Trade (dollars)	1,864,286
Finance, Insurance, and Real Estate (dollars)	1,435,294
Other Services (dollars)	1,539,337
Construction and Mining (dollars)	774,079

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[a] Seasonal activities are not shown because the large number of constraints to which the model was subject caused the water supply activities in any period to be inconsistent with the levels of water using activities in that period.

Results of Application II

The results of the initial solution of application II are shown in Tables 10a and 10b. The data which this solution represents are those discussed in Chapter VI. In this situation, the water supply is not exhausted; the constraining resource is the capacity of the waste water treatment activity. Since a portion of the water supply remains unused, its shadow price is zero. Optimum allocation in this case is that allocation which allows each water user to use water up to the point where the value of marginal product of water in that use becomes zero. In Chapter III, it was shown that this is the amount of water which each user will require if the water is free, as it is in this case. Allocation in this situation is not critical, except for the possibility of waste, wherein a water user's production function becomes horizontal at its maximum water used in this type of process will never have a negative vmp, which would discourage further use of the resource, and there is no loss to the producer if he continues to withdraw.

In order to find the point where water supplies, which flow at a relatively constant rate in application II, become scarce the requirements for water must be increased. This is done by increasing the value of all constraint parameters except those in the water supply sector (water supply itself, water treatment capacity, and waste water treatment capacity). In this case, the appropriate parameter values were doubled. In addition, as discussed in the section dealing with application I, two real disposal activities were included in the matrix. One is associated with water treatment capacity, the other with waste water treatment capacity. The effect of these two activities in the optimum solution is to indicate by how much the capacities of these treatment facilities must be increased to accommodate the higher water requirements. Such information would be of use in planning the need for capital expenditures in water supply facilities.

Table 10a. Initial optimum solution of application II (original constraints)

Objective function value: $18,997,170.20

Activity	Optimum Level	Reduced Revenue (dollars)
Pork Slaughter (carcasses)	644,800	0
Other Food and Kindred Products (dollars)	0	666.54
Other Non-durable Goods (dollars)	0	1,236.84
Farm Machinery (dollars)	0	521.62
Other Machinery (dollars)	3,066,753	0
Other Durable Goods (dollars)	0	707.19
Regulated Industries (dollars)	15,926	0
Wholesale & Retail Trade (dollars)	2,845,722	0
Finance, Insurance, and Real Estate (dollars)	2,338,714	0
Other Services (dollars)	1,713,830	0
Construction and Mining (dollars)	0	7,157.70
Water Treatment (gallons)	263,484,232 gal.	0
Waste Water Treatment (gallons)	180,000,000 gal.	0
Residential Use (gallons)	133,431,000 gal.	0

Table 10b. Initial optimum solution of application II (original constraints)

Resource	Level of Resource Use	Unused Resource	Shadow Price
Labor (workers)	1,635	379	0
Aquifer (gallons)	417,591,432	1,954,908,568	0
Pork Slaughter Capacity (carcasses)	644,800	0	13.13
Manufacturing Capital (dollars)	2,669,952	0	1.03
Regulated Industries Capital (dollars)	52,030	3,844,086	0
Wholesale & Retail Trade capital (dollars)	2,438,733	0	1.09
Finance, Insurance and Real Estate Capital (dollars)	3,991,953	0	0.57
Other Services Capital (dollars)	2,182,688	0	0.71
Construction and Mining Capital (dollars)	0	616,640	0
Water Treatment Capacity (gallons)	263,484,232	101,515,768	0
Waste Water Treatment Capacity (gallons)	180,000,000	0	0.009

Table 11. Revised resource parameters, application II

	Resource	Parameter Value
b_1	Aquifer (gallons)	2,372,500,000
b_2	Labor (workers)	4,028
$\bar{X}_1$	Pork Slaughter (carcasses) Capacity	1,289,600
$\bar{X}_2$	Manufacturing (dollars) Capital	5,339,904
$\bar{X}_3$	Regulated Industries (dollars) Capital	7,792,232
$\bar{X}_4$	Wholesale & Retail (dollars) Trade Capital	4,877,466
$\bar{X}_5$	Finance, Insurance, (dollars) and Real Estate Capital	7,983,906
$\bar{X}_6$	Other Services (dollars) Capital	4,365,376
$\bar{X}_7$	Construction and (dollars) Mining Capital	1,233,280
$\bar{X}_8$	Water Treatment (gallons) Capacity	365,000,000
$\bar{X}_9$	Waste Water (gallons) Treatment Capacity	180,000,000
$\bar{X}_{10}$	Residential Use (gallons)	266,862,000

A time horizon can be roughly estimated over which the aquifer under study will be sufficient to meet the needs of activities represented in application II. According to projections by Maki (57, Table 4, p. 8), population in the middle Missouri River basin area of Iowa, where this aquifer is located, is expected to double by the year 2020. Given a constant rate of participation in the labor force, constant production coefficients, and constant ratios of capital stock among the activities in the model, it will be at least fifty years before the supply of water in this aquifer becomes critical.

The increased values of the constraints are shown in Table 11 and the solution of the model using these constraints is shown in table 12a and 12b. A water shortage now exists; consequently, the water resource has a positive shadow price. This solution establishes a base against which comparisons can be made, assessing the possible operation of the permit system in this situation.

Note that in neither optimum solution is labor a scarce resource. This indicates that, given the existing capital - labor ratios and capital stocks in each activity, there is excess labor relative to capital as an input. Regardless of the availability of other resources, such as land or water, labor will always be in excess in this model. The inconsistency between the estimates of capital stock and labor can likely be traced to inconsistencies among the several sources from which the estimates were drawn. In any actual application of the model, it would be necessary to resolve these differences in data wherever possible, so that the productive potential of any activity will not be underestimated.

It would seem intuitively correct that if all resources except water were doubled in value, the activity levels and the value of the objective function would also double. This is not the case, however, in application II. The value of the objective function more than doubled, from approximately \$19 million to approximately \$41 million. Also, the mix of activities changed significantly, with several activities

Table 12a. Initial optimum activity levels, application II, using revised constraint parameters

Objective function value: $41,081,687.60

Activity	Optimum Level	Reduced Revenue (dollars)
Pork Slaughter (carcasses)	1,289,600	0
Other Food and Kindred (dollars) Products	0	882.59
Other Non-durable (dollars) Goods	0	628.75
Farm Machinery (dollars)	0	123.22
Other Machinery	6,133,506	0
Other Durable (dollars) Goods	0	228.64
Regulated Industries (dollars)	2,385,223	0
Wholesale and Retail (dollars) Trade	5,691,445	0
Finance, Insurance (dollars) and Real Estate	4,677,428	0
Other Services (dollars)	3,427,661	0
Construction and Mining (dollars)	1,833,977	0
Water Treatment (gallons)	365,000,000	-0.00079
Waste Water Treatment (gallons)	180,000,000	-0.00079
Residential Use (gallons)	266,862,000	0.00079
Additional Water Treatment Capacity Required (gallons)	1,699,285,600	
Additional Waste Water Treatment Capacity Required (gallons)	2,055,370,640	

Table 12b. Initial optimum levels of resource use, application II, using revised constraint parameters

Resource	Level of Resource Use	Used Resource	Shadow Price (dollars)
Labor (workers)	3,721	307	0
Aquifer (gallons)	2,372,500,000	0	0.00079
Pork Slaughter Capacity (Carcasses)	1,289,600	0	12.94
Manufacturing Capital (dollars)	5,339,904	0	1.13
Regulated Industries Capital (dollars)	7,792,232	0	0.18
Wholesale & Retail Trade (dollars)	4,877,466	0	1.16
Finance, Insurance, & Real Estate Capital (dollars)	7,983,906	0	0.58
Other Services Capital (dollars)	4,365,376	0	0.76
Construction and Mining Capital (dollars)	928,055	305,225	0
Water Treatment Capacity (gallons)	365,000,000	0	0.00079
Waste Water Treatment Capacity (gallons)	180,000,000	0	0.00079

increasing by large amounts given the increased resource parameters. These disporportionate increases are due in part to the existence of a completely new set of resource constraints and in part to the introduction of the disposal activities described above, which eliminated the constraining effect of waste water treatment capacity.

Having obtained the baseline optimal solution of application II, another solution was generated in which each production activity was forced to equal a particular value. This value approximates, within the limitations of the data, the actual relative rates of production in each activity in 1960, the year represented by the data in the model. These rates are projected into 2020 A.D., in accordance with the population projection discussed earlier. It is necessary to project the rates in order to insure that water used will have a positive shadow price. The fixed bounds on output were calculated by allowing each producing activity, with one exception, to use labor at the same rate as in 1960, as indicated in Table 18, Appendix A. The single exception is the pork slaughter activity, which was forced to operate at maximum capacity, since this is the rate of output indicated by personal interview with packing plant officials. In the case of the Manufacturing sector indicated in Table 16, Appendix A, it was necessary to apportion the labor force among the sectors, Other Food & Kindred Products, Other Non-durable Goods, Farm Machinery, and Other Durable Goods. Table 13 shows the labor force distribution and the values of the output bounds for application II. The 702 workers in the manufacturing sector were distributed among the activities on the basis of that activity's share of total employment in manufacturing in Iowa in 1960 (66, Table 33, p. 131).

In order to avoid infeasibility in the second solution, it was necessary to compare the bounds calculated above with the maximum production which the given capital stock would allow in any activity. In those activities where the use of all allocated labor was not possible because of the capital constraint, the lower output was used as an output

Table 13. Distribution of labor force among producing activities; output bounds used in application II

Activity	Labor used (workers)	Output with Maximum labor use[a]	Maximum output[b]
Pork slaughter	160	1,289,600[c] carcasses	644,800 carcasses
Other Food & Kindred Products	124	$ 943,000	$ 603,800[c]
Other Non-durable Goods	154	$1,150,000	$ 866,400[c]
Farm Machinery	92	$ 638,200[c]	$ 744,600
Other Machinery	142	$ 893,600[c]	$1,252,800
Regulated Industries	256	$3,076,800	$2,385,800[c]
Wholesale and Retail Trade	1,238	$5,876,000	$5,798,000[c]
Finance, Insurance and Real Estate	246	$4,784,200	$4,679,800[c]
Other Services	1,066	$5,264,600	$3,430,000[c]
Construction	376	$2,634,600	$2,437,200[c]
Other Durable Goods	186	$1,379,000[c]	$1,380,200

[a] This output would result if all the labor allocated to any activity were used.

[b] This is the maximum output which the given capital stock will allow.

[c] These output limits were used as output bounds in the second solution.

bound. This allows the linear program to proceed to a feasible optimal solution, in which all output bounds are satisfied.

Having specified the value of each producing activity, only the water supply vectors are allowed to change. However, the shadow price of water and the objective function value in this situation are the items of primary interest. The optimum solution given using these constraints is shown in Tables 14a and 14b.

A serious modification of the model was required in the second solution of application II. The total water use by all activities bound at the given levels is more than the total annual supply of water available from the aquifer source. Such a situation could easily arise in reality, since the aquifer parameter represents the maximum safe yield of the aquifer; an overdraft may be incurred, but damage to the aquifer would likely result. To account for this additional water requirement, a disposal activity was included which corresponds to the aquifer resource and which shows how much water would have to be withdrawn beyond the maximum safe yield. By first solving the model with the disposal activity unbounded, and then solving again with the activity bounded at the level given by the previous solution, a positive shadow price can be found which represents the vmp of a unit of water supplied either by incurring a further overdraft or by developing a new supply. A direct comparison can still be made between solutions, but it must be remembered that the solutions differ not only with respect to the value of the objective function and the vmp of water, but also with respect to the quantity of water used.

Comparison of the two solutions shows that although more water is used in the second solution than the first, the value of the objective function is smaller. This indicates clearly that the greatest return to the scarce water supply is not being realized in the projected allocation. It can be seen that optimal allocation will increase the total value added in the area by more than $1 million, and that the rate of water use can also be made significantly lower, avoiding an overdraft of the aquifer.

Table 14a. Optimum activity levels second solution of application II

Activity	Optimum Level	Reduced Revenue[a]
Pork Slaughter (carcasses)	1,289,600	-13,130.00
Other Food and Kindred Products (dollars)	603,800	- 1,000.00
Other Non-durable Goods (dollars)	866,400	- 1,000.00
Farm Machinery (dollars)	638,200	- 1,000.00
Other Machinery (dollars)	893,600	- 1,000.00
Other Durable Goods (dollars)	1,379,000	- 1,000.00
Regulated Industries (dollars)	2,385,800	- 1,000.00
Wholesale and Retail Trade (dollars)	5,798,000	- 1,000.00
Finance, Insurance and Real Estate (dollars)	4,679,800	- 1,000.00
Other Sources (dollars)	3,430,000	- 1,000.00
Construction and Mining (dollars)	2,437,200	- 1,000.00
Residential Uses (gallons)	266,862,000	0.00079

[a] Negative Reduced Revenue values indicate that the objective function value would increase if any activity with a negative coefficient were increased.

Table 14b. Optimum levels of resource use, second solution of application II

Objective function value: $39,919,070.00

Resource	Resource Used	Unused Resource	Shadow Price (dollars)
Labor (workers)	3,442	586	0
Aquifer (gallons)	2,372,500,000	0	0.00079
Pork Slaughter Capacity (carcasses)	1,289,600	0	12.94
Manufacturing Capital (dollars)	4,922,735	417,169	0
Regulated Industries Capital (dollars)	7,792,232	0	0.18
Wholesale and Retail Trade Capital (dollars)	4,877,466		1.16
Finance, Insurance & Real Estate Capital (dollars)	7,982,906		0.58
Other Services Capital (dollars)	4,365,176		0.76
Construction and Mining Capital (dollars)	1,233,671	99,609	0
Water Treatment Capacity (gallons)	365,000,000		0.00079
Waste Water Treatment Capacity (gallons)	180,000,000		0.00079
Additional Water required (aquifer overdraft -gallons)	241,022,003		

The shadow price of water in this application remains constant between the two solutions, reinforcing the earlier conclusion that the shadow price of water is apparently stable over some range of possible allocations.

The point was made in Chapter III that it is not possible to specify how the permit system will react to a water shortage. However, it can be concluded from the analysis presented in this chapter that the existence of an optimizing mechanism within the permit system would allow that system to achieve significant increases in the returns to water in a given area.

Limitations of the Model

Each application of the model developed in this study possesess certain characteristics and flaws which limit the applicability of the conclusion drawn above. Explicit mention of these limitations of the analysis is necessary in order to place the study in its proper perspective. In application I, a hypothetical situation was created so that water allocation among diverse alternative uses could be examined. The hypothetical situation required that average data be used in calculating the production coefficients for each activity, and in drawing the parameters of the model. This average data, such as that determined by using sector aggregates as activities, may not be representative of the production function of any single component of the average, such as a single firm in one of the sectors of the model. Furthermore, the reliability of many of the estimates cannot be determined empirically, since these estimates are not based upon any statistical sampling technique.

The linear nature of the production functions used in this study for each producing activity is the source of two difficulties. The first difficulty became evident above in selecting those activities which would become part of the final solution in each application. This solution is intended to approximate the permit system allocation, in that each water user in the solution is given water up to the point of maximum use. However, the point of maximum use was defined in Chapter III as the point where total physical product becomes a maximum with respect to continued water inputs. Such a maximum point does not exist in a linear production function. Output continues to increase as water use increases until another resource becomes constraining; in both applications, the constraining resource is capital stock.

The second difficulty is related closely to the first, described above. The linear production functions used in this study allow for any producing activity, no substitution between inputs, which may not be representative of the true production function. Thus, any optimum position determined using a set of linear functions may be different from the true optimum position.

One way in which both the problems described above can be accommodated is to consider more than one alternative production process for each activity. As the number of alternative processes, which can be thought of as planar approximations of the production function service. considered increases, the accuracy with which the production function is represented also increases. Such an increase in accuracy would also lend reliability to the optimum solution of the model.

Another flaw in both applications is the representation of labor as a completely homogeneous, mobile resource. This over simplification relaxes in the model constraints of immobility or of shortage of critical skills which may be important in an actual situation. Any constraints present in an actual situation and absent in the model of that situation will cause the value of the objective function to be overstated. Thus, in the short run, the maximum value of product possible if all scarce resources are optimally allocated may be impossible to achieve.

Use of the same aggregate sectors was required in application II, for although this application described an existing situation, no secondary data were available which specifically described the producing activities in the town under consideration. This application is therefore subject to the same limitations described above for application I. In addition, the problem of defining the size of the labor force on which a small, rural community can draw is a study in itself, inasmuch as it is difficult to define the geographical limits within which an available labor force may reside. For this reason, the labor resource represented in the model may constrain the producing activities at an artificially high or low level.

It is because of these limitations that these two applications serve best to demonstrate the methodology which the model represents. Values determined in application, such as the value of marginal product of water, may not be valid in other situations, and should be applied in other situations only with caution.

Summary and Conclusions

This study was undertaken with three objectives, each of which has been met with some success. The first objective, to determine by analysis of Iowa's water permit system how the system would allocate water in times of scarcity, was accomplished in Chapter III. It was shown in that chapter that the permit system acknowledges only two consistently identified points on a water user's production function, the point of zero output and zero water use and the point of maximum total physical product, where the marginal physical product of water becomes zero. The second objective was the construction of a model which would show optimum water use in particular situations. This model is discussed in Chapters V and VI. The accomplishment of these first two objectives enabled partial accomplishment of the third.

The third objective was to compare optimal water allocation and permit system allocation in a particular situation. Such a comparison was described earlier in this chapter. However, as was noted in the discussion of this comparison, it is not possible to predict what allocation will result from operation of the permit system; it is possible only to estimate limits between which permit system allocations might range, given certain assumptions concerning waste and total water use.

Related to all three of these objectives is the hypothesis developed in Chapter IV. The hypothesis states that Iowa's permit system will optimally allocate scarce water resources. It is possible to say, as illustrated by the results described previously in this chapter, that although the permit system might allocate water optimally, it is also likely that it will not. The hypothesis is therefore rejected on the grounds that no systematic bias toward finding optimum allocations can be presumed in the permit system. Note that the strict alternative hypothesis, that Iowa's permit system will not optimally allocate a scarce water resource, cannot be accepted without modification. An acceptable alternative hypothesis is that the permit system will not always allocate scarce water resources optimally.

Having reached the objectives of the study, certain conclusions can be drawn which are of perhaps greater import than the rejection of the hypothesis. The permit system in Iowa cannot be relied on to optimally allocate scarce water resource without modification; this thesis proposes a model which can be the instrument of such a modification. No vast change in the present permit system is required. It is necessary, however, that two further objectives be accomplished. First, data must be generated which will allow this model to be more accurate in describing small area water use problems. Second, a system must be devised whereby solutions of the model can be obtained simply by transferring from the data bank to the model the activity vectors appropriate to the situation under study. This system, if accurate, specific data were on call, would provide timely information in the form of priority lists among relevant activities, to those responsible for permit allocation decisions. The conclusion to which these suggestions point is that it would be possible to decrease the economic uncertainty of permit allocation by minimizing the element of randomness resulting from lack of information.

Using the model would also assist the State of Iowa in finding that allocation which will provide the greatest feasible return to the state's water resources.

The form taken by the required data referred to above is critical. The discussion of the model in Chapter V pointed out that a trade-off exists between greater detail in information and greater awkwardness of computation as the model and its data requirements grow. The suggested data set should then be composed of information which describe the characteristics of the model's activities and constraints as accurately as ease of manipulation will allow.

Such information can be envisioned with little difficulty. For example, any one of the aggregate producing sectors used in this study could be further broken down into a number of smaller, more homogeneous industry types. Linear production functions for these industry types could be estimated by sampling among them. It appears that the data so estimated could be allowed a large error tolerance, since the previous analysis in this chapter has shown that relatively small changes in the shadow price of water occur with large changes in water use. This insensitivity implies that the opportunity cost of inaccurate data may not be high.

The development of a system for utilizing this data would not be inordinately difficult. Linear programming routines have been developed and can be made an integral part of any computer installation. Data files could be established in some form of computer storage, such as magnetic tape or magnetic disc, and the required coefficients would be a part of the model's solution system.

It is apparent, therefore, that a model such as the one developed in this thesis, when utilized with the appropriate data, could be of continuing value in the administration of Iowa's permit system. It is not improbable that a model and data system such as the one suggested here could be easily maintained and updated once the data files had been established, thus providing an analytical tool which could be used to good purpose in more efficiently administering Iowa's water resources.

Suggestions for Further Research

As the conclusions of this study indicate, further research in at least four specific directions is required in order that the model developed in this thesis can be of maximum usefulness to those who are responsible for water quality management in Iowa. The first, and most urgent, direction is the development of more accurate information from which the coefficients and parameters of the model can be estimated. A data bank could be developed, in which production information could be stored. This information would be more specific than that derived from the aggregate sectors used herein. The increased specificity could come through subdividing sectors into a number of more narrow industry types and sampling within those types to derive more complete and representative descriptions of these production functions. With this data on file, a decision maker faced with determining a question of water allocation could utilize the model simply by withdrawing from the data banks those activities involved in the allocation.

The second direction of research and a necessary extension of the model is the simultaneous consideration of waste quality and water allocation questions, for these two dimensions of the water resource are highly interdependent. Accommodating quality consideration in the model requires the addition of activities which describe the changes in water use resulting from changes in quality. A mathematical simulation might also be constructed in any given water use situation which would provide the exogenous data describing changes in water quality in a hydrologic system. The simulation model would provide the linear program with data to describe changes in the hydrologic system which had taken place during the time period which the linear program represents. In this way, a constantly changing system can be represented, and the optimum water use found after any change has taken place.

Research could also be conducted in a third area. Models which describe the hydrologic system under consideration can be linked with this study's linear programming model, which describes the economic system making use of the water. In this way, changes in water resource parameters could be determined as changes in the hydrologic system took place, either as a result of water use or of changes in water supply.

A fourth area of study is indicated by the following facts. The value of marginal product of a unit of water is also the share of product which accrues to water as an input to production. This value represents the marginal cost to the State of Iowa in surrendering water for use. The marginal benefit from use, however, is being realized privately, and marginal private cost is zero since only the $15 application fee is charged for water used. This divergence between private and social marginal cost could be rectified if a fee were charged, equal to value of marginal product, for water use. This fee would also be an aid in allocation, since, in perfect competition, it represents the market clearing price of water. On the basis of what has been shown in this study with regard to the permit system and water allocation, collection of such a fee is justified. However, the assumption of perfect competition and homogeneous water supplies relied on in this thesis must be relaxed and the resulting conclusions studied prior to any recommendation on the structure of a system of fees for water use.

Whichever of these three directions of research is taken, it is apparent that this type of water resources research is an interdisciplinary field of endeavor. The inherently hybrid nature of the tools which will be needed for water resource management in the future require that research efforts be conducted in the points of intersection of economics with such disciplines as physical sciences, law, and engineering.

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APPENDIX A

Table 15. Fertilization rates and expected crop yields on soil types used in application I

Soil type	Corn		Soybeans	
	yield[a] (bushels/acre)	fertilization rate[b] (lb./acre)	yield[a] (bushels/acre)	fertilization rate[b] (lb./acre)
I. Tama silty				
clay loam	98		34	
Fertilizer:				
Nitrogen		100		0
Phosphorus		18		18
Potassium		15		0
II. Clarion loam	90		29	
Fertilizer:				
Nitrogen		80		0
Phosphorus		36		26
Potassium		15		35

[a] (50, Table 1.10, p. 15)

[b] (50, Table 1.9, p. 14)

Table 16. Rainfall and runoff by time period for three levels of rainfall

	Rainfall[a] (in)	Runoff (in)[b]	Runoff (gallons per square mile drainage area)
Average rainfall			
annual	32.12	6.572	62,105,173.4
period 1	9.37	2.74	47,639,466.8
period 2	8.60	2.188	38,042,026.7
period 3	7.46	0.874	15,195,946.7
period 4	6.69	0.768	13,352,960.0
Below normal rainfall			
annual	21.47	2.172	37,763,840.1
period 1	6.06	0.906	15,752,320.0
period 2	6.08	0.732	12,727,040.0
period 3	4.90	0.289	5,024,746.7
period 4	4.43	0.254	4,416,213.3
Above normal rainfall			
annual	37.79	10.02	174,214,400.3
period 1	11.11	4.18	72,676,266.8
period 2	10.52	3.34	58,071,466.8
period 3	8.89	1.33	23,124,266.7
period 4	7.27	1.17	20,342,400.0

[a] (80, p. 6)

[b] log (annual runoff) = -3.1 + 2.6 log (annual rainfall):
runoff in period 1: 41.7% of annual total;
period 2: 33.3% of annual total;
period 3: 13.3% of annual total;
period 4: 11.7% of annual total.
See Bennion (6, p.11).

Table 17. Crop water requirements by time period for three levels of rainfall

	Water required[a] (gallons/acre)	Rainfall[b] (gallons/acre)		
		below normal	average	above normal
Period 1:		16,463.2	25,455.5	30,182.5
Corn I	13,276.5			
Corn II	13,276.5			
Soybeans I	13,276.5			
Soybeans II	13,276.5			
Period 2:		16,517.5	23,363.6	28,579.7
Corn I	19,361.9			
Corn II	19,361.9			
Soybeans I	19,361.9			
Soybeans II	19,361.9			
Period 3:		13,311.8	20,266.6	24,151.5
Corn I	29,139.3			
Corn II	29,139.3			
Soybeans I	29,139.3			
Soybeans II	29,139.3			
Period 4:		12,035.0	18,174.7	19,750.4
Corn I	13.754.6			
Corn II	13.754.6			
Soybeans I	13,754.6			
Soybeans II	13,754.6			

[a] Based on data given in Shaw, et al. (80a).

[b] Based on data given in Shaw, (80).

[c] For any time period, supplemental irrigation required = water required - rainfall, negative irrigation requirements, implying an abundance of rainfall relative to crop use, are considered as zero.

[d] Based on data in James (51). See Table 1, supra.

Supplemental irrigation required[c] (gallons/acre)			Assumed yield[d] (bushels/acre)	Supplemental water requirements per bushel		
below normal	average	above normal		below normal	average	normal
0	0	0	98	0	0	0
0	0	0	90	0	0	0
0	0	0	34	0	0	0
0	0	0	29	0	0	0
2,844.3	0	0	98	29.0	0	0
2,844.3	0	0	90	31.6	0	0
2,844.3	0	0	34	83.7	0	0
2,844.3	0	0	29	98.1	0	0
15,827.5	8,872.7	4,987.8	98	161.5	90.1	50.9
15,827.5	8,872.7	4,987.8	90	175.9	98.1	55.4
15,827.5	8,872.7	4,987.8	34	465.5	259.6	146.7
15,827.5	8,872.7	4,987.8	29	545.8	304.4	172.0
1,719.6	0	0	98	17.6	0	0
1,719.6	0	0	90	19.1	0	0
1,719.6	0	0	34	50.6	0	0
1,719.6	0	0	29	59.3	0	0

Table 18. Definition of aggragate sectors by Standard Industrial Classification code

Sector[a]	Standard Industrial Classification codes included
1. Livestock agriculture	
2. Crop agriculture	
3. Meat products	201
4. Other food and kindred products	20 (except 201)
5. Other non-durables	22,23, 26 - 31
6. Farm machinery	352
7. Other machinery	35 (except 352), 36
8. Other durables	19, 24, 25, 32 - 34, 37 - 39
9. Regulated industries	40, 42, 44 - 47, 481, 482, 49
10. Wholesale and retail trade	50 - 59
11. Finance, insurance, and real estate	60 - 67
12. Other services	70 - 89 (except public education), 483,0722
13. Construction and mining	15 - 17, 12, 14

[a] (66, Table 1, 8 - 32)

Table 19. 1960 capital-output, output per worker, and capital-labor ratios by sector

	Sector	1960 capital per dollar output[a]	1960 output per worker[b]	1960 capital per per worker[c]	1960 employed[d]
1.	Livestock agriculture	0.6295	15,775	\$ 9,930.4	140,394
2.	Crop agriculture	1.6114	16,274	\$26,233.9	75,473
3.	Meat products	0.1423	52,455	\$ 7,464.3	27,313
4.	Other food and kindred products	0.3497	32,898	\$11,504.4	29,731
5.	Other non-durables	0.5389	15,766	\$ 8,496.3	36,999
6.	Farm machinery	0.4150	15,838	\$ 6,572.8	22,060
7.	Other machinery	0.4945	11,361	\$ 5,618.0	34,133
8.	Other durables	0.5015	12,745	\$ 6,391.6	44,259
9.	Regulated industry	2.2621	13,456	\$30,438.8	66,016
10.	Wholesale and retail trade	0.6523	5,817	\$ 3,794.4	203,648
11.	Finance, Insurance, and real estate	1.0471	30,995	\$32,454.9	37,492
12.	Other services.	0.9451	4,333	\$ 4,095.1	161,906
13.	Constructing & mining	0.1909	17,102	\$ 3,280.0	56,770

[a] (3, Table 8, p. 53)

[b] (66, Table 29, p. 127)

[c] $\text{Capital}/_{\text{dollars of output}} \times \text{output}/_{\text{worker}} = \text{capital}/_{\text{worker}}$.

[d] (66, Table 31, p. 129)

Table 20. Direct purchases and imports per dollar of gross output by sector, 1960

Sector	Direct purchases per dollar of gross output[a]	Imports per dollar of gross output[b]	Total materials cost per dollar of gross output
1. Livestock agriculture	0.364713	0.174749	0.539462
2. Crop agriculture	0.540987	0.001549	0.542536
3. Meat products	0.131098	0.000902	0.132000
4. Other food and kindred products	0.231319	0.008886	0.240205
5. Other non-durables	0.411381	0.016417	0.427798
6. Farm machinery	0.560502	0.121950	0.682452
7. Other machinery	0.639259	0.061198	0.700457
8. Other durables	0.588157	0.090549	0.686706
9. Regulated industries	0.689875	0.011813	0.701688
10. Wholesale and retail trade	0.765675	0.004516	0.770191
11. Finance, insurance,	0.613211	0.004679	0.617890
12. Other services	0.745066	0.002981	0.748047
13. Construction and mining	0.421439	0.044192	0.465631

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[a] (66, Table 26, p. 124)

[b] (3, Table 22, pp. 95-96)

Table 21. Capital per worker and estimated capital stock by major industry groups, application I

Major industry group[a]	Activities included	Capital[b] worker	Estimated employment[c]	Capital stock
Non-durable goods manufacturing	X_8, X_9	$ 3,373.9	128	$ 431,859.2
Durable goods manufacturing	X_{10}, X_{11}, X_{12}	$ 3,186.8	116	$ 369,668.8
Regulated industries	X_{13}	$30,438.8	123	$ 3,743,972.4
Wholesale and retail trade	X_{14}	$ 3,794.4	438	$ 1,661,947.2
Finance, insurance, and real estate	X_{15}	$32,454.9	62	$ 2,012,203.8
Other services	X_{16}	$ 4,095.1	439	$ 1,797,748.9
Construction and mining	X_{17}	$ 3,280.0	121	$ 396,880.0

[a] These major industry groups are defined in U. S. Census of Population (96, Table 70, p. 17-199).

[b] $\text{Capital}/_{\text{Worker}} = \text{Capital}/_{\text{Output}} \times \text{Output}/_{\text{Worker}}$. Capital-output ratio from Barnard (3, Table 8, p. 53); output per worker from MacMillan (66, Table 29, p. 127).

[c] Total model employment was allocated among major industry groups in the same proportions in which total state employment is divided among the same major industry groups in urban places of 2,500 to 10,000 population.

APPENDIX B

Table 22. Actual 1960 county and municipal employment, by sector and estimated 1967 county and municipal employment, by sector

Major industry group	1960[a] County employment	1960[b] Municipal employment	Percentage of county employment located in municipality	1967[c] Estimated county employment	1967[d] Estimated municipal employment
Agriculture	2,368	85	3.6	2,002	72
Manufacturing	213	120	56.3	448	351
Regulated Industries	251	121	48.2	273	128
Wholesale and Retail Trade	1,000	544	54.4	1,112	619
Finance, Insurance, and Real Estate	163	102	62.6	195	123
Services	971	385	39.6	1,326	533
Construction	292	150	51.4	351	188
Total					2,014

[a] (96, Table 85, p. 17-268)

[b] (96, Table 81, p. 17-234)

[c] Dr. Marvin Julius, Department of Economics, Iowa State University, Ames, Iowa. Data from a study in progress of employment and output in Iowa counties. June, 1969.

[d] Municipal employment by sector is assumed to be in the same proportion to total municipal employment as county employment by sector is to total county employment.

Table 23. Capital per worker and estimated 1967 capital stock, by major industry groups

Major industry group	Capital per worker[a]	Estimated 1967 employment[b]	Estimated 1967 capital stock
Agriculture	$ 15,620.3	72	$ 1,124,662
Manufacturing	$ 7,606.7	351	$ 2,669,952
Regulated Industries	$ 30,428.8	128	$ 3,896,166
Wholesale Retain Trade	$ 3,794.4	619	$ 2,348,734
Finance, Insurance and Real Estate	$ 32,454.9	123	$ 3,991,953
Services	$ 4,095.1	533	$ 2,182,688
Construction	$ 3,280.0	188	$ 616,640

[a] $\text{Capital}/\text{Worker} = \text{Capital}/\text{Output} \times \text{Output}/\text{Worker}$. Capital-output ratio from Barnard (3, Table 8, p. 53); output per worker from McMillan (66, Table 29, p. 127).

[b] For sources and derivation, see Table 22, Appendix B .